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MATHEMATICS IN THE PUBLIC AND PRIVATE SECONDARY SCHOOLS OF THE UNITED STATES

INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS

THE AMERICAN REPORT

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APPENDIX.

The following reports relate to institutions which, though not secondary schools exclusively, cover more or less of the secondary field in their work:

a. Report on Mathematics in Evening Technical Schools.

By A. D. Dean, State Education Department, Albany, N. Y.

b. Report on Private Correspondence Schools.

By W. F. Rocheleau, Interstate School of Correspondence, Chicago, Ill.

c. Report on Mathematics in Schools and Colleges for Negroes.

W. T. B. Williams, Hampton Institute, Hampton, Va.

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MATHEMATICS IN THE PUBLIC AND PRIVATE SECONDARY SCHOOLS OF THE UNITED STATES.

COMMITTEE NO. III. PUBLIC GENERAL SECONDARY SCHOOLS.

I. GENERAL REPORT.

ORGANIZATION.

The pupils.—The public secondary schools of the United States, usually called high schools, receive pupils that have completed an elementary school course, generally of eight years. The age of pupils beginning high-school work is about 14 years.

The studies of the elementary school include arithmetic, geography, history, English grammar, literature and composition in English, with other studies that vary according to the location of the schools and with the resources of the communities that support them. These studies do not include foreign languages, except in very rare instances, and only occasionally include rudimentary study of algebra,[1] the construction or measurement of geometrical drawings or models, or the study of geometrical facts beyond a few rules for mensuration of simple plane figures and the very simplest solids. The algorithm for square root is often taught, but that for cube root seldom. The greatest emphasis and the longest practice are put upon commercial applications of arithmetic.

The purpose of the high school.—The purpose of high-school education has generally been twofold—not only to furnish preparation for college, but also to provide some of the elements of a liberal education for those whose formal and directed study is to go no further. In not a few communities high schools were established in which the former purpose was expressly waived; but even in these schools the gradual broadening of college entrance requirements has combined with the pressure of able and ambitious pupils to establish college preparation as a recognized purpose of high-school work. More recently a new type of high schools has been devised, to furnish preparation as direct as possible for certain special vocations, notably those of the machinist and the merchant. These will be dealt with in another report. This report is confined to high schools of the general type.

[1] The word "algebra" in this report includes all arithmetic in which letters are used to represent numbers.

The length of the course.—The high-school course is generally one of four years. The work is usually done in a separate building from the elementary school, by more highly paid teachers. In thinly settled and poor communities the work is frequently done in the same building as the elementary school, and even by teachers who must give part of their time to the care and teaching of the younger grades. In such cases the period of high-school work is shortened sometimes to two or even to one year, and obviously can serve only for the beginning of college preparation, if for any; the pupil must go to some more central institution to complete the high-school work.

Until within a few years the high-school diploma in the city of Boston was given for three years' work, a postgraduate year being necessary for the usual college preparation. At present this is no longer true in large centers.

In a very few cases the work of the high school has been carried into subjects usually reserved for college; this, however, can not be said to present a tendency of actual development to-day.

The six-year high school.—The last few years, however, have seen a decided pressure for the extension of high-school work downward into the two upper grades of the elementary school. The six-year high school thus constituted had its most telling argument doubtless from the language teaching, in earlier utilization of the imitative faculty; it presents, however, great advantages for mathematics, especially in the possibility of closer correlation of arithmetic and algebra and in the introduction of intuitional geometry at an appropriate age.

As this proposition has been detailed for consideration in the city of New York, the six-year high school would be divided into separately organized parts, so that the complete school career of a pupil would consist of six years in the elementary school, three years in the lower high school, and three years in the upper high school. In some other places it has taken the form of departmental organization of the last two years of the elementary school under the direction of the high-school department heads. In five high schools, which include the two oldest schools in America and a school in one of the youngest States in the Union, pupils are admitted at the end of their sixth school year and the six years' work is organized as one whole.

A separate report on six-year high schools has been prepared by a subcommittee under the chairmanship of Prof. A. S. Gale.

Supervision.—In about a dozen States of this country there is no supervision of high schools exercised throughout the State, either by the government or by any university. In these States the high schools are under the direction of the local school board and its executive officer, the superintendent. The results are not chaotic, in spite of the lack of centralization; for the superintendents and the

teachers under them confer with their fellows elsewhere and follow the lead of universities, though not officially related to them, and of such associations as the National Education Association and their several committees.

Even in the States where the high-school course of study is prescribed by the government or by the State university some latitude is generally left within which the local authorities can modify the work, especially for pupils not destined for the university.

The elective system.—In some high schools the pupils are permitted to choose, with the consent of their parents and under the advice of their teachers, not only the general aim of their work but the separate courses of instruction that seem to contribute to that aim, just as a graduate student does in a university. A diploma is then given for the completion of a certain number of units of work—generally not less than the equivalent of 20 courses, each consisting of 4 hours class instruction per week for 1 year.

Election by courses.—In most high schools, however, an attempt is made to provide for varying aims of study by organizing groups of studies, with descriptive titles, such as "The classical course," "The English course," "The commercial course," and so on. The State of Maine officially recognizes seven such courses in the regular high schools, besides providing for agricultural high schools as separate institutions.

State regulation.—Such regulation as that just mentioned is based upon State aid to the schools, or upon the admission of their graduates to State normal schools or State universities. Communities receive grants of money from the State treasury on condition that certain conditions of equipment are met and that the courses of study suggested by the State authorities are efficiently maintained. In certain cases State university officials inspect the schools to see that the instruction is of satisfactory quality.

In many States this dictation and supervision is confined to careful definition of the subjects of study and of the time that may be allotted to them, leaving the grouping into "courses" to be done by the local authorities. This is in line with the plans now in vogue for defining requirements for admission to college—in which many more subjects are provided for than any one student can possibly think it wise to take, so that individual preferences and aptitudes can be accommodated, and the total achievement estimated by a uniform though somewhat artificial standard of replaceable educational values. Among studies which all pupils are expected to include in their work are, generally, English language and literature, history, and 2 years of mathematics, including plane geometry and algebra.

The minimum requirement.—Where the State authorities define in detail the various "courses" of study among which the pupil

may choose, algebra and plane geometry generally appear in all such courses. This is the case even in such specialized courses as those labeled "agricultural" or "commercial," but Maine excepts the "household arts course," in which there is only a year of "mathematics" not otherwise described.

Throughout the country there is an apparent minimum of over 3 years of mathematics required;[1] this includes, however, a year or so that might properly be classed as elementary school work, being arithmetic drill of a purely mechanical type, studied solely for the sake of its commercial applications.

The principal.—The principal or executive head of a high school gives his whole time to administrative duties in the largest schools. As we pass to smaller and smaller schools we find that the principal must devote more and more of his time to teaching, until in small towns the high-school principal has only a short time each day for the duties peculiar to his position. More time is often secured for them by making him also superintendent of schools, or by combining all the schools into one institution with kindergarten, elementary, and high-school departments.

In the absence of general laws regulating the high-school studies throughout the State, the principal of a small school has great influence in changing the character and extent of the work done, sometimes even to the extent of completely changing the aims and fortunes of the school. An energetic man in this position can always materially affect the methods of teaching and the details of the material covered, if the subject is one upon which his opinion is definitely formed.

The head of the department of mathematics.—Where two or more are teaching mathematics in one school, the custom is growing of having one teacher designated as head of the department. Sometimes he is selected by the principal, more often by the superintendent of schools or by the school board. His duties include a more or less definite responsibility for the work in mathematics throughout the school, especially with a view to the coordination of parallel courses; he conducts department meetings, confers with the principal, informs himself as to textbooks and teachers available for future work. His advice or direction is sometimes effective upon the topics of instruction, generally effective upon the methods.

Cooperation.—The body of teachers, in a high school separately organized (i. e., not in the same building with the elementary school or not having the superintendent of schools as principal), has little or no influence upon the teaching of mathematics in the elementary school. A considerable influence, on the other hand, is exerted upon high-school mathematics by the existence of college entrance

[1] Mr. E. E. Whitford, in School Science and Mathematics, April, 1910.

standards. Within a few years an additional influence has been the organization of associations of secondary school teachers of mathematics and of kindred subjects, in which college instructors have participated. Less than half of the high schools of the country are represented in these associations, and their effect seems rather to have been the awakening of interest on the part of the teachers than actual reform on the part either of subject matter or of method.

The teachers.—The best of the high-school teachers of mathematics, with rare exceptions, have no more knowledge of mathematics than would be indicated by a 1 year's course in the calculus. The great majority of them have not even that. Standards are, however, improving, the principal obstacle being ignorance and indifference on the part of educational authorities. This obstacle is more obvious in States where there is no State university at the head of the educational system, or where there is no State-wide organization of education. The city superintendent of schools is generally more an organizer than a scholar, and represents the demand for elementary education rather than for secondary, for practical education rather than for college preparation; and thus far mathematics in high schools continues to point its signboards toward the university. As a whole, then, the result is a lack of emphasis on preparation of teachers of mathematics. In the smaller high schools it is not uncommon to find the bright young girl who was studying solid geometry last year teaching algebra or geometry this.

Racial restriction.—Throughout the South negroes are excluded from the high schools where white pupils are taught. In certain localities separate high schools are provided for the negroes; work done in them is, with very few exceptions, of a distinctly inferior character.

Promotions.—In about 75 per cent of the high schools promotions are annual, in the rest semiannual. In the South and in the smaller schools promotion is more often by class; elsewhere by subject.

THE CURRICULUM.

Directive influences.—The curriculum in mathematics is determined in general by the admission requirements of colleges. This is, of course, confessedly so in the States in which there is a complete and State-wide organization of education, with the university recognized as the final stage. It is also true in other States, and even in the smaller communities where few, if any, of the pupils may be planning to go to college, and where the local school committee disclaim the intention of following university guidance; and it is true of the mathematical curriculum even where it is not true of other subjects of study. The reason for this nearly universal dependence on college definitions

is that mathematics is not otherwise defined by any authority that the schools feel willing to accept.

The definite and legal enactment of all public high-school courses of study is in all cases made by the school committee (often called the school board, or the trustees). Only occasionally does this body contain members able or willing to decide upon details of a subject that seems to the ordinary layman so abstract as mathematics. Where the definitions of the university are not accepted bodily, the advice of the principal is generally sought, occasionally that of the head of the department, or of the special teacher of mathematics.

Importance of the textbook.—There are States in which the textbooks are prescribed; in one all the textbooks of mathematics are prescribed as those of a certain author, reliance being placed on the publishers to keep his productions up to date. Some of the teachers replying to the questions of the committee state that no deviations from the textbooks are allowed; but this is doubtless the decree of the local authorities rather than of the State, and is undoubtedly due to the unpleasant experience of trying to connect the work of a radical experimenter with that of a successor incapable of appreciating and pursuing a wide and uncharted departure from the orthodox course.

Correlation of mathematical subjects with each other.—The different branches of high-school mathematics are not in general correlated with each other, but are pursued one after the other with such differences of method and of point of view that algebra is often forgotten by the time geometry is completed. Of recent years the teachers in a small number of schools have, of their own accord and with considerable difficulty, arranged the geometry work so that algebra can be applied to some of the numerical problems given in illustration of the metrical theorems. More than anything else the subject of proportion is treated algebraically. In a still smaller number of high schools there is a well-organized blending of the different subjects, algebra, geometry, and trigonometry, into a general and progressive course in mathematics.

The customary independence of these mathematical subjects is restricted almost everywhere by the requirement that a pupil must have "passed algebra" before he is permitted to begin geometry.

Correlation with science.—The mathematics work is little correlated with physics, though many schools insist on algebra at least, and sometimes geometry or even trigonometry, before beginning physics. The requirement of algebra for chemistry students also is not unknown.[1]

[1] An ideal state of things is described by Miss Thirmuthis Brookman, as the result of 12 years' development in the high school of Lincoln, Nebr. Here an "inspirational" half-year course in general science serves as an introduction to a course in mathematics, where each week's work is "a definite and clear-cut section of a well-proportioned system," including algebra, geometry, and trigonometry. Pupils showing marked inability in the general science introduction are not urged to enter the work in mathematics.—School Review, January, 1910.

THE SUBJECT MATTER.

Elementary algebra and plane geometry.—Every regular high school in the United States offers algebra and plane geometry for at least one year each. Half of them give algebra for an extra half year; less than 20 per cent give algebra for two full years. A very few schools give algebra for two years and a half, and a very few give plane geometry for a year and a half.

Solid geometry, trigonometry, and college algebra.—Solid geometry for a half year, plane trigonometry (often with spherical right triangles, occasionally with the general spherical triangle[1]) for a half year, and advanced algebra, so called, including certain special topics listed below, for a half year, are given in some of the larger high schools or in some of the smaller ones that definitely prepare for college.

The textbook as evidence.—The fact that few of the high-school teachers of mathematics are thoroughly trained in their subject and that the subject matter is settled for the most part without their initiative indicates that the content of the curriculum will be closely defined by the textbooks used. This is also the comment of publishing houses in discussing new textbook projects: "We must have a book that the ordinary teacher can follow without change; it is only the exceptional teacher that can strike out independently of its guidance."

The kind of textbook in general use up to 12 or 15 years ago, and the kind most widely used to-day, will enable us to define the subject matter and the methods of algebra, geometry, and trigonometry as presented to more than 75 per cent of the high-school pupils of the United States. Hereon is based the following outline.[2]

THE ORTHODOX SYLLABUS.

I. Algebra to Quadratics.

Introductory.—Definitions and "axioms," discussion of negative quantities, brief practice in algebraic expression and interpretation, one or two lessons in the use of algebra for problems so simple that algebra adds to their difficulty.

The four operations.—Addition, subtraction, multiplication, and division, completed successively in that order, with formal rules of manipulation (not necessarily stated in advance); literal and fractional coefficients and exponents used; ingeniously involved parentheses, brackets, and braces; and expressions sometimes more complicated than most of the pupils will ever see again.

[1] More than a half year is taken in this case.

[2] For the purpose of this outline only, "numbers" will be understood as expressed *without letters*, and "problems" will be understood as "clothed" in words.

Factors.—Factoring expressions, such as the difference of two squares, ax^2+bx+c, $x^n \pm y^n$ (often with "demonstrations," as of the case where the sign is + and n is odd); factoring "by parts;" forms like $x^4+x^2y^2+y^4$; expressions such as can be obtained from the simpler forms by substituting binomials for one or more of the letters. Usually no application is made of these feats except in the reduction of fractions, and in highest common factor and least common multiple, which follow as introductory to fractions.

Highest common factor and lowest common multiple.—Highest common factor, first by factoring, then by the Euclidean method. A demonstration is usually given for this, but is hardly ever assimilated by the pupils. Not seldom the proof given is applicable only to numbers—that is, it will not hold for literal expressions in which, as is usual, some of the dividends have to be multiplied and some monomial factors have to be saved out. Lowest common multiple, generally by factoring only, with a perfunctory comment on the method which utilizes the highest common factor.

Fractions.—Fractions, with the rules of transformation formally demonstrated, and the four operations each completed in its turn. Expressions of ingenious complexity are handled under each head.

Simple equations and problems.—"Simple" equations, that is, equations of the first degree with one unknown letter, and abounding in parentheses and fractions; and, at last, problems to be solved by means of such equations, except that the equations needed for the purpose are really simple.

Linear elimination and problems.—Two-letter linear equations, including fractional and literal equations; three-letter equations of the first degree, and occasionally simple four or five letter sets. Equations solved for the reciprocals of the letters involved. Problems leading to equations of the first degree in two or more letters. Literal equations are scattered at random under this topic and the preceding one.

Inference of equations.—The model examples worked out in the textbook generally indicate the inference of an equation from the preceding work by means of phrases printed at the side, such as "transposing," "clearing of fractions," "adding to eliminate the x terms," etc. No expectation is indicated that the pupil will use any substitute for these annotations.

Neglect of practice in devising equations.—The number of problems given under this topic and the preceding one is generally insufficient to give real facility in algebraic expression, and their introduction seems isolated, an interruption in the progress of manipulation. Haste or neglect at this point is explained by the fact that no topic is more difficult to test adequately in a written examination than the

devising of equations, and where time is scant it will be devoted to topics that show.

Involution and evolution.—Under the title "Involution" next are treated powers of numbers and of monomials, and squares and cubes of binomials; under the title "Evolution" are treated square roots and cube roots of polynomials. Some details of the theory of exponents are necessarily included under these heads.

Radicals and radical equations.—Radicals, including the rationalization of binomial denominators, and the square root of a binomial surd (generally given without adequate demonstration); radical equations, carefully selected or constructed so as to give, upon rationalization, equations of the first degree. Extraneous solutions, sometimes declared admissible because of the "ambiguous" sign of the square root. Problems again, few in number, leading with suitable choice of letters to radical equations.

Exponents.—Theory of exponents, without any mention of logarithms; good correlation with the preceding topic.

II. Quadratics and Beyond.

Quadratics in one and two unknowns.—Quadratics in one unknown, first without the second term ("pure" quadratics) and then complete quadratics ("affected" quadratics); problems leading to such quadratics. Linear-quadratic pairs, elimination by substitution; special cases of quadratic pairs solved by devices suited to each case; special emphasis on symmetrical equations, solved by reducing to values for $x+y$ and $x-y$. Very few problems. Literal equations at random under this topic.

Ratio and proportion.—Ratio and proportion, including the traditional transformations of a proportion; examples of literal equations, and of problems to which proportion can be applied if one insists; no mention of its application in geometry, and no comment on the relation of this subject to fractional equations. This topic is not referred to under any other part of the work in algebra.

The progressions.—Arithmetical progression; formulas for the nth term and for the sum of the terms, any three of the five constants being given, to find the other two.

Geometrical progression; formulas for the nth term and for the sum of the terms, certain groups of three of the five constants being given to find the other two constants. Formula for the "sum of the series" when the ratio is less than one and the number of terms indefinitely great; recurring decimals.

Inserting means.—Arithmetic and geometric mean; insertion of two or more arithmetic or geometric means between two given numbers.

Binomial theorem.—The binomial theorem for positive integral exponents; proof of the same;[1] application to powers of a binomial of which the terms may be complicated with fractions, with radical signs or with exponents that may or may not be positive or integral. Formula by which any term of any power may be written down.

III. Plane Geometry.

The five books.—The sequence in all the ordinary textbooks is that of Legendre; five books, the first on lines, angles, triangles, and other polygons; the second on circles and the measurement of angles; the third on proportion (treated as an algebraic subject) and similar figures; the fourth on areas; the fifth on regular polygons and the measurement of the circle.

Incommensurables.—Incommensurable ratios occur in Books II, III, IV, and V; in most schools an attempt is made at every one of these points to master the explanations given in the book.

Construction.—Geometrical constructions, to be made as with Euclid by the use of the compass and unmarked straightedge, are given in a logical place among the other propositions. Locus theorems are given, beginning with the bisectors of angles and the perpendicular bisectors of lines in Book I.

Original exercises ("riders").—Exercises are given for practice in the invention of demonstrations similar to those in the text, and in the application of available theorems to numerical data. These exercises reach the number of 500 or 600, and are often accompanied by suggestive notes or diagrams. They are for the most part rigidly confined to subject matter like that in the text, so that successful practice with them does not add materially to the geometrical information of the student.

IV. Solid Geometry.

Order of topics.—This includes successively the following topics: Perpendicular and parallel lines and planes; diedral, triedral and polyedral angles (including the ratio of incommensurable diedrals); equivalence and congruence among, and the measurement of, parallelopipeds, prisms, pyramids, cylinders, and cones; the geometry of great-circle diagrams on a spherical surface, with scant reference to the corresponding polyedral angles at the center of the sphere; the surface and volume of the sphere.

Mensuration theorems.—Development is almost never used for the lateral areas of cones and cylinders, and the assumption that there is a quantity spoken of as "the area" of a cylinder, cone, or a sphere is tacitly made (a similar assumption having been made for the "length" of the circumference in plane geometry), the areas of successively

[1] In most classes this proof is omitted in teaching.

approximating figures approaching this assumed quantity as a limit. The important distinction between area and volume in this respect is not commented upon.

Method of limits.—There are about a dozen places in plane and solid geometry in which the method of limits is used to deal with the incommensurable numbers which have arisen in the work. In nearly all cases the proofs are so cast that a variable number is determined in two different ways independent of each other, and use is made of the following "Theorem of Limits":

"If two variables are equal and each approaches a limit, the limits are equal."

Pupils seem to be able to learn to repeat the words of this theorem while failing in many cases to appreciate the cogency of the proof in which it is used. The treatment of limits in geometry is the source of much discontent not only among the teachers themselves, but also among the university examiners and professors.

V. Trigonometry.

Introductory.—Definition of the functions of an acute angle; generalization to angles in any quadrant. Representation of the functions by lines drawn on a unit circle; change in the values of functions from one quadrant to another.

Formulas.—Proof of the formulas for the sine and cosine of the sum and of the difference of two angles; of the tangent of the sum and of the difference; of the sine, cosine, and tangent of the double and the half of an angle; formulas for the transformation of the sum or the difference of two sines or two cosines into products; practice in the use of all these formulas in reductions.

Logarithms.—Generally logarithms are studied at this point.

Triangles.—Application of logarithms to computation of a right triangle; proofs of formulas for an oblique triangle, using trigonometric algebra as much as possible. Simple applications to surveying and navigation.

VI. Advanced Algebra.

Topics treated.—Under this head are given various disconnected topics including: Theory of quadratic equations, with graphs of $y=ax^2+bx+c$; solution of numerical equations of higher degree in one unknown, with graphical illustration; occasionally successive derivatives of algebraic polynomials, geometrically interpreted, are incidentally taken up as far as advisable in utilizing graphs for explanation; occasionally trigonometric solutions are given for certain equations. Choice and chance. Determinants, with practice in reduction and evaluation (the multiplication theorem omitted). Indeterminate coefficients.

Purpose.—The sole purpose in this course seems to be to furnish information that may be useful in later mathematical study.

Confusion of title.—In many schools the latter part of the course in elementary algebra described previously is styled "Advanced algebra." This usage is confusing and should be avoided.

VII. Arithmetic.

In addition to the subjects which are generally recognized as secondary school studies, many high schools give a half year in arithmetic. In smaller schools and in districts where the elementary schools are not so effective, either on account of short terms or on account of more recent establishment of public school education, this is given at the beginning of high school work; in other schools it is given after the years devoted to algebra and geometry, and is called "Advanced arithmetic." The topics are not in any respect different from those treated in elementary schools, though the problems are somewhat more difficult and aim at a closer correlation with commercial practice. There is a decided purpose to attain facility and accuracy in routine operations. Little or no effort is made to treat arithmetic as a science, or as having any real connection with the other mathematical subjects in high school work.

GROWTH OF THE COLLEGE REQUIREMENT.

Harvard College as Example.

Advance of the last 60 years.—The definition which we have just completed of the subjects of mathematics covered in high school work represents a very considerable advance over the conditions 60 years ago. This change can be well traced by a study of the requirements for admission to Harvard College.

Early textbooks.—In 1845 Lacroix's Arithmetic, Euler's Algebra, Davies's First Lessons and his Introduction to Geometry "to VII of Proportions" and "Algebra to the Extent of Square Root" were required for admission; this was a larger requirement than was made at that time by any other New England college.

In 1866 Chase's Common School Arithmetic, Sherwin's Common School Algebra "to Section XXXVIII," and Hill's Second Book in Geometry, parts I and II, were required. The algebra requirements so defined omitted radicals and fractional exponents, proportions, and algebraic and geometric progressions. The scope of the geometry requirement is considerably less than that of the subject of plane geometry to-day; it is developed in the textbook referred to by first acquainting the pupil with the facts of geometry, and then discussing the methods of proof, together with some suggestion of how proofs can be arrived at.

The next year the metric system was added to the arithmetic requirement, and algebra was defined as "through quadratics." No

reference was made to a particular textbook for these two subjects, but elementary plane geometry was defined by the "first 13 chapters of Pierce's Treatise." In the next year the practical use of logarithms (not the theory) was advised, and was required the following year.

Elective requirements.—In 1870, while the old subjects of examination were retained for such as chose to take them, an alternative specification was made with a much reduced requirement in Latin and Greek and an increased requirement in mathematics. This included permutations, probability, determinants, in algebra; solid geometry; theory and use of logarithms; plane trigonometry "by the analytic method"; elementary mechanics and hydrostatics.

For some years the experiment was tried of accepting plane analytic geometry as an examination subject; but it seems to have been on the whole unsatisfactory, and within the last 10 or 12 years it has not been included among the alternative requirements for admission.

Changes in subject matter.—The textbooks by Thomas Hill, referred to above, contained a few numerical exercises and "originals" (riders), and did not rigidly limit the subject by the compass-and-ruler canon; Davies's Legendre, which was much more widely used, was merely Euclid rearranged; it was studied more often as a classic document than as a scientific textbook; and it contained no originals. Chauvenet's geometry, published about 1870, contained good originals, not very carefully graded, 281 of them on the five "books" of plane geometry. It was not until 1875, and then only for an "optional" examination, that originals in geometry appeared among the requirements for admission to Harvard College; for this the exercises in Chauvenet's geometry were recommended as affording a good preparation.

In the textbooks of algebra 50 years ago much more stress was placed on logical exposition than on the solution of problems. The development of arithmetic, as followed in the textbooks of the elementary school, was faithfully imitated in algebra, and various "operations" (some, like the square and cube root of polynomials, having no conceivable use, and others mere pedantic elaborations of methods that in simpler form were well worth while) were laboriously discussed and exploited before the use of equations in discussing problems was entered upon.

The algebras of Todhunter and Hamblin Smith, in England, were followed in America by Wentworth's Algebra, published about 1881. The exercises in the latter book, which were widely commended by the teachers of the day as "well chosen, numerous, and carefully graded," were the selling feature, and the book sprang into great popularity at once. The idea was to "learn by doing," and since that time the exercises have been of much greater importance in textbooks and in teaching than before.

Causes of change.—The changes thus noted in the school work of algebra and geometry are the result of attempts to take subjects that were originally a part of the college curriculum and adapt them to the comprehension of high-school pupils. Abstract discussion was to be replaced by means for practical drill; the teacher, rather than the mathematical scholar, was the arbiter in choosing books. At first he rejected books like Hill's and Sherwin's, which were actually better adapted to his classes, for those which had been familiar to him in his own work in college. From those the progress was gradual. Ten years ago a prospectus of the two books last referred to would have seemed quite up to date.

RECENT PROGRESS.

The last 10 years.—In the last 10 years changes in the details of high-school mathematics have been radical and rapid.[1] They are largely the result of the active interest taken in the work of high-school teachers by university professors, and of the conference between high-school men of different localities. Both of these influences have made themselves felt through teachers' associations. So far as the changes and tendencies referred to have actually begun to affect teaching, they appear in recently published textbooks in good use. For this reason the committee has examined some thirty of the recent high-school books on algebra, geometry, and trigonometry, and presents the results of that examination here.

The order of topics.—In almost all these books geometry is supposed to follow a year's work in algebra, though there are one or two books, whose success is still not completely assured, which essay a combination of the two subjects. A combination (or "blending") of plane and solid geometry does not seem to have been seriously attempted.

In algebra the order of topics is only slightly varied from the following:

1. Introduction, negative numbers, etc.
2. "The four operations."
3. Factors, H. C. F. and L. C. M. by factoring.
4. Fractions.
5. Simple equations and problems.
6. Elimination, linear systems.
7. Powers and roots, exponents, radicals.
8. Quadratic equations.
9. Elimination of quadratics.
10. Literal equations, generalization.
11. Proportion, "the progressions," logarithms.
12. The binomial theorem.

[1] See "Present Tendencies in the Teaching of Geometry" (in the United States) in A. W. Stamper's History of the Teaching of Geometry, New York, Columbia University (1906).

In geometry the order of development is still mostly that of Legendre, the five books of plane geometry being successively polygons, circles, similar figures, areas, and regular polygons; and the solid-geometry order being planes and lines, polyhedral angles, prisms and pyramids, and the "three round bodies." Two books transpose the third and fourth books of plane geometry; here and there also is shown a disposition to group propositions in smaller lists than the "books," without, however, changing the order much.

The textbooks in trigonometry agree fairly well on the topics which should be taken, but differ widely in their estimate of the relative importance of these topics.

A common arrangement of topics is the following: Functions of an acute angle; solution of right triangles by natural functions and by logarithms; functions of any angle; general value of an angle; the addition and subtraction formulas; formulas for double an angle and half an angle; the conversion formulas; and the solution of oblique triangle by logarithms. In addition to these topics, we find the radian measure, inverse functions, and the line representation of functions, and, in some books, graphical discussion of functions and a careful treatment of the measurement of angles near 0° and 90°.

Graphical methods of discussion.—For some 15 years there has been increasing pressure for the introduction of Cartesian coordinates as an instrument of study in elementary algebra. It began to appear in the schoolbooks about 1898. All but one of the books here examined make use of this device; sometimes it is given in a separate chapter, in one case in an appendix. In a few others it is made an effective part of the structure of the subject. The word "function" is sometimes used, but even without it the work generally begins by plotting curves in which y is a non-algebraic function of x, so that the student gets some insight into the functional relation.

The graph of a two-letter linear equation is pointed out as a straight line. No proof is given, though one book remarks that the "proof follows easily from the geometry of similar triangles." Elimination of linear pairs is illustrated, generally also linear-quadratic and quadratic pairs. No comment is made, as a general thing, on the limitation of this illustration to two-letter equations.

In about one-half of the books the solution of a numerical quadratic equation by the use of a standard parabola ($y = x^2$) and a straight edge is mentioned, and its use recommended as a check on the solution obtained by the algebraic process.

Computation.—In spite of the fact that John Perry's propositions for reform in mathematical teaching have been widely and on the whole favorably considered in this country, not one of the textbooks of algebra, and only one of those in geometry, makes any reference to the number of significant figures in a number as a criterion of the

degree of approximation. None gives any directions for economical methods of computation having regard to the degree of accuracy warranted by the data, or gives problems with data appropriate for such practice.

In a geometry textbook of excellent character and long use, problems appear in which the data are such as would occur under the actual circumstances described in the problem; but no directions were given for dealing economically with the difficulties of arithmetic thus introduced, and the problems were considered unsuitable by the schools. In the more modern books here examined the data are all of the predigested sort (one or two-figure integers).

In trigonometry the subject of approximate computation in general, and the fact that only approximate results can be obtained by the use of trigonometric functions, are rarely referred to.

Checks.—The practice of checking the solution of an equation in algebra by substituting the roots, and checking an algebraic transformation by substituting arbitrary values, appears in almost all the algebra textbooks. In trigonometry the subject of checks for the solution of triangles is not carefully treated—several books give no checks except the obvious one for the angles when the sides of the triangle are given. The topic does not appear at all in geometry. No reference is made anywhere to methods of checking the details of computation; it being assumed, no doubt, that the matter has been adequately treated in the study of arithmetic. The textbooks in that subject, and the irresponsible habits of the pupils in computing, do not furnish good ground for such an assumption.

Notation.—In algebra there is a tendency to break the monopoly that the letter x has had in representing numbers whose value is sought. The symbol $\neq$ for "is not equal to" and the symbol $\equiv$ for algebraic identity have come into use. In geometry the symbol $\equiv$ is apparently favored for congruence, instead of $\cong$, which already had good authority. For the exceedingly mysterious thing which some authors call the "intrinsic" sign of a number, a diminutive plus or minus sign is sometimes used.

The notation of lower-case letters for lines and for lengths of lines has at last been introduced into elementary geometry textbooks, to the great advantage of the algebraic proofs. This appears in three of these books, one of which applies it very imperfectly. The same book also uses lower-case letters to represent the number of degrees in an angle, though analogy to the unlimited and the limited straight lines would suggest using a capital for both the point and its associated magnitude, the angle.

Logical terms.—The technical terms of logic are mostly avoided. The term "reductio ad absurdum" occurs in three; the same thing is called "indirect proof" in four others; the "method of exclusion" is

pointed out in three.* "Reductio ad absurdum" and the "method of exclusion" are both classified as "indirect proof" in one book, while "indirect proof" and the "method of exclusion" are both classified as "reductio ad absurdum" in another. One book manages to avoid all mention of these terms.

The logical inverse is called the converse in all current textbooks, and there is no exception in these latest books. The obverse is so called in one book, is called the opposite in two others, and is omitted in the rest. The contraposite is so called in one, is called, curiously, the contradictory in another, and is not mentioned in the rest, although one book points out the equivalence of the converse and the opposite.

The term "immediate inference" is given and much used in one book. Homothetic position is used for the similarity theorems in two of these textbooks, but under different names ("radially situated," "in perspective"), and in only one of them is it systematically used as a means of demonstration.

Innovations.—The changes in the subject matter of algebra have been referred to under preceding heads. Those in geometry are much less extensive, no doubt because of a widespread belief in the invulnerability of the logical structure represented by the successively dependent propositions. The treatment of incommensurable magnitudes in geometry seems, in particular, to be protected by sacred tradition.

The theorem of limits, namely, that "if two variables are equal and each approaches a limit, the limits are equal," is used systematically in all but two of the books examined, and one of these puts it in an appendix.

The similarity theorems are based on the area theorems in two. Axial and central symmetry are both used in three, and neither appears alone. The idea of symmetry is used for comment and illustration rather than as a method of attack or as a resource in argument.

A decided innovation is the custom of explaining methods of attack for new theorems; this is contained in almost all of the newer textbooks.

The word "congruent" is used in four books, one of the others using the old-fashioned phrase, "equal in all respects." Only one of the books using the word "congruent" uses the word "equal" in the sense of "equivalent."

In solid geometry also there are few innovations—that is, innovations so far as really popular use is concerned. Among them are shaded figures or photographs of actual models, the spherical degree (or "spherid" as one book calls it) as the unit of area on the sphere, and the prismoidal formula. None of the books speaks of a "unit of solid angle;" it is always a "unit of area on the sphere." Some,

even of the most enterprising of these authors, adhere to the trirectangular triangle as a unit. The prismoidal formula is apparently not introduced as a means of simplifying the logical structure; it is rather a new addition to the old task.

Problems in algebra.—Much more space is given to equations, and to problems giving rise to equations, as a response to the frequently repeated contention of teachers, uttered in magazine articles and in teachers' associations, that the equation should be the fundamental work at least for the first year. In most cases, however, the preparatory study of transformations (multiplication and division, factoring, fractions, etc.) is carried to a degree far beyond what is necessary for the manipulation of any reasonably probable equations, certainly beyond what are given during first-year work. The study presents, therefore, a somewhat disconnected aspect—first, transformations treated in a systematic and fairly complete fashion, with a few intrusive illustrations of the application of them to equations that do not need much; then, equations treated as material for practice of a small part of this manipulation; and, finally, problems, hopefully sought from newer quarries, designed to show how equations might arise that could be managed by this manipulative skill.

It would probably be easy for a young person of good judgment, engaged in reviewing his high-school algebra, to learn that problems cause the invention of equations, and that transformations are necessary for the solution of equations. The prominence given, however, to this systematic development of what must be considered the mechanical side of algebra tends to weaken the interest of the pupil in that part of the subject that is of most value, not only to him whose education stops with the high school, but surely also with the future student of engineering or of pure mathematics; the study, that is, of expressing the conditions of actuality in mathematical form, and of interpreting mathematical results in terms of time, space, and things.

The problems in all these books are very plentiful; sometimes the teacher is warned that the work should not include the solution of *all* the problems, and that their profusion is his opportunity to vary his work from class to class and to specify fresh problems for review lessons. While the character of the problems in three of the books is strictly orthodox, the others take their data freely from geometry, physics, and even from engineering and dietetics (1). There is no hesitation in utilizing the properties of similar triangles, or the phenomena of falling bodies; but there is a chaste reluctance to do anything more than mention the existence of reasons at the back of the facts and formulas used. Our old friend the clock problem survives the dead and buried hare and hound, and the solution in odd elevenths of a second continues to pass without challenge.

Long division, square root, and generally cube root of algebraic expressions are still treated at length, without any comment on the futility of the pupil's attempt when the result does not "come out even." [1]

The comment that an extraneous root of a radical equation can be made to satisfy the equation "by suitably choosing between the two possible signs of a square root" is passing out of use.

Problems in geometry.—The number of "original" exercises to which the pupil is expected to apply his newly acquired knowledge varies from 600 to 1,200 in plane geometry, and from 300 to 600 in solid. Some of them use freely the results of the modern geometry of the triangle and of projective geometry, but without any introduction of the corresponding modern methods in the text. One book, however, includes the nine-point circle, the radical axis, and the notion of reciprocal theorems in the regular text, but omits all mention of symmetry.

Some effort is made to create a "practical" atmosphere, and one or two modern books are rich in allusions to and illustrations of such things as surveying, parquetry, and architecture. They are dealt with, however, by the sometimes cumbrous methods of Euclidean geometry, instead of by such means as practical men would use.

The fundamental definitions of trigonometry appear, and in a couple of books the tables of natural functions; in one there are two-place tables, in the other a four-place, while in both is used what seems to be a three-figure angle (degrees, and tenths by interpolation).

Loci.—The definition of a locus is introduced at the very beginning in one textbook; in Book II in another; in most of them it appears in Book I in connection with the theorems about perpendiculars and bisectors. In no textbook does the locus of an algebraic equation in two variables appear, though this would seem a not inadvisable corollary of the introduction of graphs into algebra. The nearest thing to it is the problem to find the locus of a point moving so that the figures determining it retain certain numerical properties, as in the locus of the vertex of a triangle with fixed base and constant perimeter. The "real" or "practical" locus problem is still rare; a type of such is the artisan's way of testing the cross-section of a cylindrical core box with a steel square.

While no reference whatever is made to the locus of an equation in the text books on geometry, some of the trigonometries use it in tracing the changes in the values of sine, tangent, etc., as the angle varies; all of them, however, make some reference to the representation of the trigonometric ratios by lines related to a unit circle. Only one comments on the important fact that here we have a line representing a pure number.

[1] See Missouri Teachers' Course in Algebra, § I, (b), § X, (c). School Science, April, 1908.

Problems in trigonometry.—The problems in trigonometry textbooks comprise the proofs of identities and the numerical solution of equations as opportunities for practice in the transformations of trigonometric expressions; and also the solutions of plane triangles, with applications to questions of surveying, civil engineering, and navigation. The pupil is expected to remember all the formulas referred to on page 21.

Spherical trigonometry is usually considered a college subject, but spherical right (and quadrantal) triangles are often dealt with in the secondary school. With these the so-called Napier's Rules furnish the possibility of memorizing formulas, apparently an indispensable requisite in subjects offered for admission examinations.

Only one of these books mentions the fact that oblique triangles may be solved by dividing them into right triangles. The idea of projection and of the angle functions as projection ratios does not enter into these books. One book refers to it, but makes little use of it.

The solution of oblique triangles comes late, following the addition and subtraction formulas. One book gives a geometric proof for the tangent formula. Angles are generally expressed in degrees, minutes, and seconds; two books have degrees, minutes, and decimals of a minute, and one has a set of examples in which degrees and decimals of a degree are used. Few books have a discussion of the theory of logarithms. It is evidently assumed that the student has received from algebra a knowledge of the nature and properties of logarithms sufficient for the practical application to the solution of triangles. The use of the augmented characteristic in the case of a negative logarithm is almost universal.

We find in one textbook the radian measurement introduced on the second page in the book and no further use made of it, while in another book it is found well along in the book, where an excellent discussion of it is given, and frequent use is made of it in equations, identities, and problems. We find a similar condition of affairs when we look for the inverse functions.

Only one of the books completes the solution of plane oblique triangles before entering upon the discussion of the standard formulas of trigonometric algebra.

EXAMINATIONS.

Before the high school.—Examinations are not generally required of pupils applying for admission to high school if they have been advanced through the regular grades of the school system up to that point. Pupils from private schools or from distant communities are in general examined for admission.

In the high school.—Examinations are required for promotion each year and for graduation. In a very large percentage of the

schools replying to our questions pupils attaining high marks (generally 90 per cent) in class work are excused from these examinations.

After the high school.—Examinations are required for passing from high school to university except for pupils certified from an "accredited" high school; that is, one of which the university has officially approved the course of study and method of teaching. This privilege is continued only to those schools whose pupils bear out in the main the favorable opinions expressed on their certificates.

Effect of examinations.—As to the effect of these several classes of examinations on pupils and teachers, diametrically opposite views are expressed, the weight of opinion being apparently favorable so far as the examinations in the high schools themselves are concerned. Various modifications of the university admission examinations are suggested, such as permitting the pupil to take all his books into the examination room and to use them as he chooses; the perfection and extension of the accrediting system is also suggested.

"Accrediting" elementary schools.—For passing from elementary schools to high schools a few suggestions were made looking toward a system similar to the certifying of high-school pupils for college, a system in which the high schools should maintain an attitude of friendly criticism toward the elementary schools, testing their work rather by its efficiency as preparation for subsequent study than by a written examination.

METHODS.

Relative popularity.—In the questionnaire issued by this committee 12 methods were listed, the names being taken from well-known books on the teaching of mathematics. The replies indicated clearly, by practice or preference, that five of these methods were most popular:

- Measurement and computation.
- Laboratory method.
- Use of models.
- Use of cross-section paper.
- Individual method.

Next in order of popularity, after a considerable interval, come the following:

- Combination of algebra and geometry.
- Out-of-door work.
- Paper folding.
- Observational geometry, to precede deductive geometry.

The least popular of the 12 suggestions offered were the combination of plane and solid geometry and of geometry and trigonometry; next to them in unpopularity, curiously enough, is the "heuristic" method; and, in view of the noticeable tendency in recent and widely adopted textbooks, the inference is unavoidable

that this apparent unpopularity is due to the fact that "heuristic" is a hard word.

On the other hand, the low position of "observational" geometry in these reports is significant. A progressive teacher in Arkansas writes: "We tried a six weeks' course in observational geometry in the second high-school year and saved time by it, but the pupils did not like it; they were too old." The most successful work in observational geometry has been in schools where it could be done by pupils of 12 years of age; the six-year high school is probably a necessary condition for the best use of this expedient.

Outdoor work and paper folding depend on local conditions and individual teachers, respectively. One high-school teacher borrows a transit from the city engineer every spring. Paper folding, used occasionally for symmetry propositions at the beginning of geometry, has not been found available for much else.

The combination of algebra and geometry has a good start in the sense that algebra is applied to the discussion of geometry problems involving numerical relations, but except in isolated cases there has been no blending or interweaving of the two subjects into one.

The five topics listed as the most popular emphasize the decided tendency to cultivate the intuitional side, to utilize sight, touch, and muscular sense as avenues to the pupils' intelligence.

Separate problem book.—One or two attempts have been made by publishers to present algebra by means of a manual and an exercise book separately, but except for review work this plan is hardly ever followed. Principles and practice appear not only in the same book, but as nearly as possible on the same page.

The use of models.—Models, in most cases made by the pupils themselves with cardboard, wire, and fine cord, or thread, are very much used in solid geometry.

The model for equivalent parallelopipeds and for the dissection of a triangle prism into three equivalent pyramids can not generally be so made, and the school owns the patterns, if it is not so fortunate as to own the right sort of teacher. The spherical blackboard is generally used.

Squared paper.—Cross-section paper is very much used, generally for the illustration of elimination, sometimes also for the introduction of the idea of function. A considerable opportunity here for the correlation of algebra and geometry seems to be entirely neglected. Little or no attempt is made to find approximate solutions graphically, in cases where elementary algebra will not help out.

Computation.—Measurement and computation are undertaken in true schoolroom style. No attempt is made to work to a specified degree of accuracy, or to acquire convenient and systematic habits of computation. Logarithms are only used in the trigonometry class, and the slide rule has not appeared in high schools.

Improvement of problems.—Search is being made everywhere for problems that make a more direct appeal to the interest of the pupil than the collections handed down by previous generations. Even in the not infrequent case where these problems are themselves highly improbable as instances of perplexity in human beings, some gain is made by clothing the inventions of the schoolmaster in the words of people that walk the streets to-day, and a great many of them are "real" problems of "practical" import.

The marks.—The marks by which pupils are ranked and promoted are most frequently based on oral recitations and on exercises written out in the classroom; written examinations at stated intervals and the teacher's general estimate of the pupil's power and achievement are also utilized for this purpose; and least often exercises written out at home or voluntary original work.

Failures.—Failures are reported as low as 3 per cent of the membership of the class and as high as 45 per cent; most of the reports are evenly distributed from 5 to 25 per cent. The State superintendent's report for New Hampshire is probably a typical result for small high schools; it contains the following details:

Results of examinations.

	Number passed.	Number failed.	Per cent failed.
First year algebra	1,666	385	19
Second year algebra [1]	364	45	11
Third year algebra [1]	668	23	3
Plane geometry, Books I and II	1,243	134	10
Plane geometry, completed	632	23	3
Solid geometry	169	7	4
Advanced arithmetic	590	95	14
Trigonometry	84	1	1

[1] Second and third year algebra are not consecutive.

All the results reported are likely to give too low an estimate of the number of failures, mostly because they are tested from the point of view of the teacher conducting the course, and with a view to deciding whether work assigned has been performed; not by ascertaining the pupil's efficiency in a subsequent course of study for which his high-school work is a necessary preparation. A special report on failures in high-school mathematics has been prepared by a subcommittee of which Mr. William Betz is chairman.

Unsatisfactory records.—The pupils' marks in different studies are generally entered on a permanent record, available for reference in subsequent years. The keeping of these records is an irksome task, involving a considerable amount of clerical labor after the actual teaching has been completed; it is often neglected. So far as furnishing data from educational experience is concerned, it is practically worthless, and will continue to be so until it is very much expanded.

Valuable details unattainable.—Any investigator consulting a high-school record would be unable to ascertain whether a pupil is proficient in numerical computation, in algebraic manipulation, in knowledge of mathematical facts, in comprehension of successive logical dependence, in mathematical invention and ingenuity. These are presumably the qualities his mathematical studies are to develop. A pupil at the head of the rank list might be presumed to be excellent in all, and one at the foot to be a failure in all, though this is not necessarily so; a pupil midway between head and foot may have conspicuous defects in some of these and conspicuous excellencies in others, and it would be extremely important to have this fact definitely known.

Educational values.—Again the various educational improvements suggested from time to time, such as the practice of checking numerical work, the use of cross-section paper, the use of a textbook without full ready-made demonstrations, may produce results that are not aimed at, which can only be discovered by the isolation of these marks. In general, such records as these would be indispensable for long-continued educational experiment, or for statistical investigation.

No device suggested.—The great majority of our replies, while admitting that such a system of records was not in use, were decidedly in favor of it, but expressed great doubt as to its feasibility. The only suggestions made for the purpose were to the effect that specially ruled class books (marking lists) would be desirable.

Mathematical museums.—Practically every high school has some mathematical instruments, models, etc., but mathematical museums comparable in any degree with that at Columbia University are very rare, even in university towns.

Mathematical clubs for pupils.—Of mathematical clubs to which the pupil has access, there are very few. There is one at the Stuyvesant High School for Boys in New York, and one is promised at the William Penn High School for Girls in Philadelphia. The McKinley High School in St. Louis reports as follows:

The Mathematical Club of the McKinley High School has been organized for three years. During the earlier part of the organization the history of elementary mathematics was studied. Topics were assigned to the different members of the club and reports were made upon these. Along with this work mathematical curiosities and puzzles were presented and discussed. The more recent work has had to do with the use of the slide rule and all kinds of graphic solutions; problems were selected that would show wide use of the slide rule, and other problems to show the value of graphic methods. Mathematical puzzles were continued.

AIMS.

General culture and college.—The aim of high-school education, so far as mathematics is concerned, is general culture, and at the same time is preparation for college. There is a slight preponderance of popularity in favor of the former reply, but the latter is close enough

to it to make it clear that in the minds of school committees and other official bodies, general culture seems to need the sort of mathematics that is prescribed for admission to college. A considerable fraction—from 25 per cent in the Middle West to 75 per cent in the Atlantic States—include preparation for technical institutes as a part of the purpose that mathematical education is to fulfill.

Not for occupation.—Of the careers that teachers are consciously looking forward to for their pupils, those of the merchant or accountant, the civil or mechanical engineer, the teacher, the farmer, and the "woman of the house" are frequently mentioned. It may well be doubted whether the mathematical training received in high schools gives added efficiency to any degree, in any of these callings; that is to say, the effect of the kind formerly acclaimed and now discredited under the name of general discipline; certainly not to any greater degree than the same amount of time and interest applied to geology, or chemistry, or even to music. The pupil learns to solve algebraic equations, but does not apply them to devising balanced rations for the farm stock out of the crop he raises; he learns about similar triangles, but knows not so much as his grandfather about the thrust on a girder, or about the way of mapping a river, or of estimating the amount of excavation in grading the house lot. He is brighter and keener, because he has been educated, and he has self-reliance because he has done things by his own thinking power; but he has had nothing directly contributory to the special knowledge or aptitude required for these occupations that he may follow—except teaching; surely he can teach what he has been taught, and he does. Thus schools inherit.

The college determines the aim of high-school work.—Since the high school does not prepare by means of mathematical teaching for occupations in which the pupils may subsequently engage, and since mathematics as deemed desirable for general culture is defined by the entrance examinations of the university, the whole question of the aim of this subject in high schools must be decided by the views of university people, and made known, aside from the scanty information of catalogues, by the character of examination questions. On this account the report on examinations will be of great interest.

The presumption is, then, that the subjects of high-school mathematics contain information and cultivate aptitudes that are necessary and useful in college mathematical work; or that so far as they do not do so, they are more useful in other ways than subjects that would do so. This is not true.

No systematic study of the desiderata.—The subject of geometry, for example, passed down from the university to the high school, continued to be taught with little change of content or method as a consequence of the transfer. There is no indication that the present

syllabus of high-school mathematics has been adjusted by university professors to the needs of their departments. Nobody knows what parts of it are absolutely necessary for subsequent work in exact science, or whether any of the necessary parts could with advantage be postponed to the university and replaced by topics of interest and of greater value to high-school pupils.

The question is not discussed in such a systematic way as that. Changes that have been brought about, mostly additions, have been fortunate guesses, proposed by tentative examination questions and afterwards reenforced by argument and formal demand. Such are the locus problem and the "original" or "rider" in geometry, the use of graphs in algebra. Within a few years some inconsiderable omissions have been made, such as the division process for H. C. F. in algebra, the square root of a binomial surd, and so on; and a decidedly critical attitude has been maintained against the traditional type of problems based on highly impractical data. All these things, while not aimless, can fairly be called tinkering rather than scientific reconstruction.

Influence of teachers' associations.—The formation of associations of mathematical teachers, in which university professors are actively interested, is bringing about a different attitude on this subject. Several of them have prepared and published reports on aims and methods in algebra and geometry. The most thorough and scientific work may be expected from the committee appointed recently by the National Education Association to form a syllabus of elementary geometry. The plans of that committee include an inquiry into all the considerations that should weigh in determining the scope of the subject, and its membership is such as to warrant the expectation of valuable results.

II. REPORTS OF SUBCOMMITTEES.

SUBCOMMITTEE 1. BOYS' HIGH SCHOOLS.

In order to obtain material for this report, questionnaires were sent to 25 boys' high schools—all that could be found—in various parts of the United States. Replies were received from 10 of these schools, and this report is a summary of the information thus received.

Though the number of schools which sent replies to the questionnaires was not large, they include some of the largest and most important schools in the United States. Of the 10 schools replying, one has 2,800 pupils; four have between 1,000 and 2,000; three, between 500 and 1,000; and two, less than 100. Two are situated in the Southern States, three in New England, two in New York, one in Pennsylvania, and two in Maryland.

ORGANIZATION.

In eight of these schools the department of mathematics is organized with a head usually nominated by the principal of the school, with the approval of the superintendent of schools. No information was obtained in regard to the general management of the department.

In six of the schools the teachers of mathematics have to teach other subjects, and in half of the schools they have work other than teaching, such as the supervision of athletics, pupils' organizations, or journalism.

The teachers of mathematics are generally members of some association of teachers of mathematics, and four of the schools report that these associations have some influence on the teaching of mathematics in the high school. It does not appear that there is any similar influence exerted by the mathematical departments on the teaching of arithmetic in the elementary schools.

The average age of pupils at entrance is about 14 years; three schools report the age as 15 years, and one as 13 years. In all the schools the available work in mathematics extends throughout the entire school course. The four schools in the South and in Maryland report racial restrictions against certain classes of pupils, presumably against the negro. Promotions in seven of these schools are annual, in three semiannual; by class in four, and by subject in six. The reports in regard to the per cent of the whole number of pupils taking mathematical courses are unreliable, owing evidently to a misunderstanding of the question.

THE MATHEMATICAL CURRICULUM.

The mathematical curriculum is generally determined by school-board courses of study, and these are influenced largely by the requirements of higher institutions of learning. Four schools report that recommendations made by the principal and approved by the superintendent have great weight in determining and modifying the curriculum.

The different branches of mathematics are correlated in six of the schools. The correlation generally means some sort of combination of algebra and geometry, or geometry and trigonometry. In nine of the schools the course in mathematics is progressive, in the sense that admission to second-year mathematics is conditioned upon the satisfactory completion of the first-year course, and so on. Mathematics is correlated with other studies in about half of these schools, and physics is usually mentioned as such a study. One school states that algebra, geometry, and trigonometry are required of pupils who take physics. Another school says that the science, drawing, and mathematics departments are working together by

furnishing problems and explanations which relate to the other departments. Six of the schools made a report on the sequence of topics in the course. Five of them begin with some work in arithmetic, and then take algebra, plane geometry, solid geometry, and trigonometry in the order mentioned. Two schools report spherical trigonometry and one analytical geometry.

EXAMINATIONS.

Usually pupils coming into these schools from the elementary schools of the town or city are admitted without examination. All but two schools require examinations for graduation. In one school pupils who attain high rank, 90 per cent or more, are excused from the examinations for graduation.

Five of these schools secure the admission of their pupils to higher institutions of learning by certificate, and five by examination.

Opinion is divided as to the effect of these examinations, though a small majority of answers seem to indicate that the teachers believe that the examinations for graduation and for admission to higher institutions have a good effect, both on the teaching of mathematics and upon the pupil's attitude toward the subject. In the case of those schools that are not favorably disposed toward these examinations, it does not appear that any attempt has been made to eliminate them.

METHODS OF TEACHING.

The laboratory method and the individual method of instruction are used in very few schools. Paper folding and outdoor work do not seem to be popular, but the use of cross-section paper is general. Observational geometry is taught in only a few schools. Some schools are trying to combine algebra and geometry, and geometry and trigonometry.

The marks by which the pupils are ranked are based most often on oral recitations, on exercises written in class, and on written examinations at stated intervals. The teacher's estimate of the pupil's power and achievement is also an important factor in determining the mark. In six schools written home work, and in eight schools voluntary home work are also used.

The per cent of failures varies from 10 per cent in one school to 40 per cent in another, but most of the schools report from 15 to 25 per cent of failures, with an average of 20 per cent for all the schools.

Very little attempt seems to have been made to devise a method of marking by which it would be possible to determine from the pupil's marks or record whether he is proficient in numerical accuracy, algebraic manipulation, knowledge of geometrical facts, comprehension of logical dependence, or mathematical invention and ingenuity. Three schools report such a method in use, and one of these is the

largest boys' high school in the United States; the other schools seem to take little interest in the subject. One report says that such a system would be useful "only for the purpose of diagnosing talent."

Only two schools, both in New York City, have access to a museum of mathematical models and instruments. One school, also in New York City, has a mathematical club where officers and members are pupils of the school.

AIMS OF MATHEMATICAL TEACHING.

All the schools report that the aim of mathematical teaching is general culture and preparation for college. It is evidently assumed that either one of these includes the other. Seven of the schools also prepare their pupils for technical schools.

SUBCOMMITTEE 2. GIRLS' HIGH SCHOOLS.

The report of subcommittee 2, on girls' high schools, is based on the results of a questionnaire prepared by the members of Committee III, and sent out by the Bureau of Education to all the secondary schools which came within the province of that committee. Replies were received from 12 girls' high and Latin schools. These reports have been tabulated and, in so far as possible, the facts and opinions elicited from individual schools on each question have been summarized. While on many points attitudes have proved so divergent that general conclusions are impossible, the replies as a whole indicate the general trend of mathematical teaching methods in this small but important group of schools.

The field of investigation undertaken in the questionnaire was divided into five main heads: Organization, curriculum, methods of examination, methods of teaching, and aims of teaching. A careful examination of the returns in these departments suggested the value of grouping replies to certain questions by the geographical sections represented, namely, Greater Boston, the Middle States and the Southern States. This plan was followed wherever there was an approach to uniformity among schools of each section combined with marked sectional differences. In certain other instances the main divergences appeared between the larger and the smaller schools. Three schools were represented in the Greater Boston group, five in the Middle States, and four in the South.

Summaries have been prepared for each of these groups and for the entire group of 12 schools. This report will concern itself, in the main, with the entire group, indicating sectional differences and individual peculiarities whenever they appear to be pronounced.

Taking up in detail the five main heads covered by the investigation, namely, (1) organization, (2) curriculum, (3) examinations, (4) teaching methods, (5) aims of teaching, the facts established and the opinions expressed have been arranged in order under these headings as follows:

ORGANIZATION.

Here the most marked divergence appears, between the three sections represented, in the matter of regulative powers attached to the office of principal. In the Middle States, the principal's power depends largely upon his personal influence; in Greater Boston he possesses general regulative power within the limits of a course of study fixed by the school board and acts in conjunction with the school board; in the South, on the other hand, the entire responsibility seems to rest with the principal. In the Pennsylvania schools the principal exercises his power in an advisory capacity, but takes formal action only by authority of the superintendent of schools. In Greater Boston the principal's activity is confined largely to making recommendations to superior officers of the school board. In the South the principal shapes the policy and guides the work of the mathematical department.

With two exceptions, and these in the case of smaller schools, the mathematical departments are organized with a head who is nominated usually by the principal, approved by the superintendent and elected by the school board. In one school, the Cambridge (Mass.) Girls' High School, the head is chosen by the teachers of the mathematical department. A typical method of management is outlined in the report of the Reading Girls' High School: "Departmental meetings are held once a week; methods, plans, etc., are discussed; class work is kept parallel; representative papers of students are submitted to and approved by the head of the department."

In the Philadelphia High School for Girls the departmental management is described as follows:

> We hold departmental meetings once a month for teachers of the upper classes. At these meetings we discuss the textbooks to be used, the eliminations to be made from the texts we are using, the ground to be covered in a given time (not as a rule less than 15 weeks), the examinations to be given at the end of the half year, the needs of the classes as determined by the maturity of the pupils and by their previous preparation; often the needs of some special individuals. * * *
>
> The head of the department, having but three classes each day, finds time to visit the other classrooms, and, especially in the case of the younger members of the department, to regulate the teaching, partly by suggestion and sometimes by taking charge of the lesson.
>
> Once during each half year we try to have a simultaneous test for the pupils taking first-year algebra and one for those taking first-year geometry. * * * The head of the department assumes the responsibility for all pupils promoted with conditions in mathematics, planning their work for them, * * * to enable them to remove the conditions as speedily and successfully as possible.

In nine schools, the teachers of mathematics, excepting the departmental head, are required to teach other subjects in order to fill out their personal programs. In three schools the work of mathematics teachers is confined, in the main, to mathematical subjects.

The schools are practically unanimous in failing to require supervision of athletics, journalism, or pupils' organizations from teachers of mathematics, but in the matter of clerical work they are about evenly divided in practice, about half of the schools requiring clerical service from the mathematics teachers.

In four of the schools an attempt has been made to cooperate with other schools in furthering special aims of pupils. The Philadelphia Girls' High School makes special provision, at the end of two years from entrance, for the transfer of girls, whose aims are thereby furthered, to technical schools. The Girls' High Schools and the Girls' Latin School of Boston transfer and receive pupils with great freedom whenever such change ministers to the vocational or college preparatory needs of the pupil.

Attempts to influence the mathematical instruction in the elementary schools are made by four of the Girls' High Schools. This influence is exerted in two ways—through mathematical associations and through the personal affiliations of teachers. The graduates of the Philadelphia High School for Girls, for example, go in large numbers to the Normal School and later become elementary-school teachers. By personal affiliations with these teachers the mathematical department of the Girls' High School of Philadelphia is able to bring considerable influence to bear upon the teaching of mathematics in the elementary schools of Philadelphia.

Exactly half of the schools report that the mathematical teaching in them is affected by the higher educational institutions. The influence thus exerted comes through the admission requirements of the normal schools, colleges, and technical schools; and through the mutual efforts of college and high-school instructors in mathematical associations and committee work. Seven of the schools are represented in such associations and six of these report that such membership has resulted in a helpful and stimulating influence on the mathematics teaching of the school.

The average age of pupils entering the schools for the first time varied from 13 to 17 years. In two schools the average was 13 years; in six, 14 years; in three, 15 years; and in one southern school 17 years. The time required for graduation is the standard four years of the regular secondary course. In the Boston Girls' Latin School a six years' course is offered, the first two years of which receive pupils who would otherwise be pursuing the two final years of the elementary-school course. The mathematical course, as a rule, covers the entire

four years, either as a required or as an elective. In four of the schools only the course is limited to three years.

Eight years is the normal time allowance assigned to the elementary course that precedes admission to the secondary-school course, though in two of the southern schools seven years only are required.

No race restriction whatever is imposed upon candidates for admission to the schools of the Northern and Middle States, but in the Southern States none but white girls are accepted for membership.

Promotions in four of the schools are semiannual, in the remainder annual; in five schools pupils are promoted by class, in four by subject, and in two by both methods.

The number of pupils registered in the schools under investigation varied from 135 to 3,629. In the 12 schools there were a total of 14,650 pupils, an average of 1,221 to each school. Reports of the per cent of pupils in the school taking mathematics of the several years were incomplete. The summary of the southern group gives the following averages: First year, 65 per cent; second year, 20 per cent; third year, 15 per cent; fourth year, 5 per cent. For the Middle States the following average is given: First year, 42 per cent; second year, 27 per cent; third year, 15 per cent; fourth year, 5 per cent for one school and 1 per cent for another school. In the Boston Girls' Latin School the full mathematical course is required from all the pupils in each year of the course.

The percentage of the total membership of the school made up of graduates varied from 3.5 per cent to 23 per cent. The average percentage of the schools reporting on this point was 13 per cent.

MATHEMATICAL CURRICULUM.

In six of the schools the mathematical curriculum is affected by the admission requirements of higher institutions; it is prescribed in the case of 10 schools by school-board courses of study, which are prepared in the case of 6 of them with the constructive assistance of the principal, acting in consultation with the head of the mathematical department. In general the means of modifying the mathematical curriculum are the same as those by which that curriculum is originally determined. The same machinery must be set in motion for revision that was employed for organization and construction. In eight of the schools an attempt is made to correlate instruction among the several branches of the mathematical curriculum. No general statement of the method employed is possible, since in all schools the correlation is informal and depends for effectiveness very largely upon the individual teacher. The subjects are taught, generally, as separate branches, but the materials furnished by one subject are employed in another. This is increasingly true of the use of applied problems, which are drawn from all the available sources.

In eight of the schools a serious effort is made to correlate the mathematical instruction with that of the naturally allied branches of science and drawing. This correlation appears principally in the use of applied problems from physics and chemistry and in special instruction in those topics of the mathematical course most fundamental to the study of these sciences. The best developed plan reported was that in use at the Reading High School, which has worked out a scheme somewhat in detail, from which the following is quoted:

Mathematics is correlated with the study of science in this school, but more especially as regards chemistry and physics. Problems in chemistry, given from time to time throughout the course, require the student's ability to make and solve simple algebraic equations. The college preparatory physics course presupposes a thorough grasp of algebra and plane geometry in order to understand the forming of the various formulæ. Continual use is also made of this knowledge in solving numerous problems throughout the course.

MATHEMATICS.	DRAWING.
The relation of numbers	applied in æsthetic drawing to the size used for a study.
Fractional division	for copying in reduced or enlarged form, and for size of component parts.
Proportion	used in reference to relation light and dark, or color; to proportion the size and color for distant objects.
Proportion	is taught in industrial drawing to emphasize fitness of size and shape for its purpose of use; for weight and tone of units in design; space relation in reference to the distribution of the finished design.
Proportion	used in mechanical and geometric work; required in the instruction for working drawings of geometric solids, etc.
Problems in mensuration applied to surfaces	used in circular work, and conic sections.
Do	used to determine the amount of material for making models.

Generally speaking, admission to a course in any other subject does not depend upon the completion of a given course in mathematics, except in so far as failure to pass in a mathematical subject would result in the loss of promotion to the next higher class. Courses are often so arranged that in order to take up a particular subject, such as physics, a pupil must have reached a specified upper class and hence must have completed a certain amount of mathematics. In this sense only is admission to a non-mathematical subject conditioned upon the completion of a course in mathematics.

In 75 per cent of the schools the mathematical course is progressive, in the sense that a given course of mathematics must have been completed as a condition precedent to a pupil's admission to the course of a subsequent year.

The following outline of a mathematical course submitted by the High School for Girls of Reading, Pa., is typical of the courses offered

by the best schools and is complete for all the subjects offered in the secondary school mathematical courses for this group:

ARITHMETIC.

- Denominate numbers.
- Percentage.
  - Simple percentage.
- Profit and loss.
  - Trade discount.
  - Interest.
    - Simple.
    - Accurate.
    - Compound.
  - Bank discount.
  - Taxes.
  - Duties.
  - Insurance.
- Ratio.
- Proportion.
- Partnership.
- Involution.
- Evolution.
- Mensuration.
- Metric system.
- Greatest common divisor.
- Least common multiple.
- Review principles preparatory to study of algebra.

Pupils are expected to solve practical problems in any one of these subjects.

ALGEBRA.

- Four fundamental operations.
- Factoring.
- Highest common factor. Least common multiple.
- Fractions.
- Linear equations: Numerical, literal.
- Problems depending on linear equations of one or more unknown quantities.
- Graphic solutions.
- Radicals, including square and cube root.
- Exponents, fractional and negative.
- Quadratic equations: Numerical, literal.
- Problems depending on quadratic equations.
- Binomial theorem for positive integral exponents.
- The formulas for the nth term, and the sum of the terms of arithmetic and geometric progressions with applications.

Throughout the course pupils are required to solve numerous problems which involve putting questions into equations. Some of these problems are chosen from mensuration, from physics, and from commercial and everyday life.

PLANE GEOMETRY.

The theorems and about 200 original exercises are studied. Special stress is laid upon construction work. About two weeks time is spent on the theory of limits. Locus problems are made interesting to the pupil, who is given frequent opportunity of applying *all* the geometric principles taught and of working out demonstrations for himself. The work is done *very* slowly at first, and but one-third of the course is covered the first term.

SOLID GEOMETRY.

The theorems and constructions of the text are studied. The solution of numerous original exercises, including locus problems, is required.

TRIGONOMETRY.

Definitions and relations of the six trigonometric functions as ratios; circular measurement of angles. Proofs of formulas, particularly for the sine and cosine and tangent of the sum and difference of two angles, of the double angle, and the half angle, the product expressions for the sum or the difference of two sines or of two cosines, etc. Solution of trigonometric equations of a simple character. Theory and use of logarithms. The solution of right and oblique triangles and practical applications.

EXAMINATIONS.

In eight of the schools examinations in mathematical subjects (mainly arithmetic) are required for admission. In practically all of the schools mathematical examinations are required for graduates who are candidates for admission to higher institutions. Many of these institutions, of course, will receive candidates on certificates of proficiency furnished by the school. Noncertificated pupils are in all cases subjected to examination. The schools are substantially unanimous in the opinion that these several classes of examinations have a beneficial effect both on the teaching of mathematics and on the pupil's attitude toward the subject. As one teacher puts it: "The effect is toward thoroughness and accuracy in work. The average student dislikes examinations but examinations do not make the student dislike mathematics."

In three schools only have important recent changes in the department of examinations been introduced. Such changes have been in the direction of exemption of pupils with a high term average from final examinations and the increase of weight given to the recitation mark over the examination record. The system of exemption from final examinations by schools and the substitution of certificates for examinations by the higher institutions are the chief proposals made for the elimination of the various kinds of examinations now in vogue. They have proved satisfactory where adopted and furnish an indication of the growing tendency to place less emphasis on the value of the single formal test furnished by the examination and to give greater weight to a continuous record covering an extended period.

METHODS OF TEACHING.

Under this heading the attempt was made to determine whether each of 12 specified methods was in use in the school making the report and also, irrespective of its adoption, whether the person making the report considered that method educationally desirable. These replies will be considered in detail, as follows:

The "laboratory method" was in use to a limited extent in a little more than one-half of the schools and was considered desirable by

more than one-half. The "heuristic method" and the "individual method" were in use in exactly one-half of the schools and were considered educationally desirable by seven schools. Little interest was manifested in the discussion of "paper folding" or "outdoor work," only two schools reporting these methods as in use and only three expressing the opinion that they were of value in teaching. Most of the schools employed models and felt that their use was justified by the results. Seven schools made use of "measurement and computation," and of "cross-section paper," while only one school preceded the study of formal deductive geometry, by "observational" geometry, although five of the schools believed this precedence to be educationally sound. Six schools teach "algebra and geometry" somewhat in combination, though in a purely informal manner and under the separate captions of algebra and geometry. Three replies indicate a similar treatment of "plane and solid geometry" and of "geometry and trigonometry."

Opinions expressed in the replies varied as to the "stage of the pupil's progress at which the emphasis should be placed upon grasp of logical relations rather than upon manipulative skill," from 10 years of age to 15 years of age. The majority of the replies, however, leaned to the ninth or the tenth school year, indicating that this transfer of emphasis should be made during either the first or the second high-school year.

The basis for marking mathematical work appeared to be as follows: All the schools mark on recitations made orally in class; only three give any considerable weight to written home work, whereas all schools make written class work an important element in determining the pupil's rank; eight schools allow credit for voluntary original work; four give weight to the teacher's general estimate of the pupil's power and accomplishment, and all the replies indicate that examinations furnish at least a partial basis in the final rank, such examinations occurring either at stated intervals or at the term's end.

Reports of the percentage of failures in mathematics from various causes varied from "percentage of final failures almost 0" to 30 p cent. The average of failures reported was 13 per cent.

While only two schools reported methods of marking from which it was possible to determine a pupil's proficiency in such details as "numerical accuracy," "algebraic manipulation," "knowledge of geometrical facts," "comprehension of logical dependence," and "mathematical ingenuity," five schools considered such a method of marking desirable. Six schools, however, were opposed to such a system and only one school came forward with a suggestion in any way practicable for registering such records of capacity, in the detail indicated by the questions. This suggestion was an outline of the

method employed by teachers in making their class reports in that high school. "Each teacher makes a report of her classes six times a year, in which she specifies whether one of the five items mentioned in which proficiency may be shown or indifference is responsible for the pupil's failure."

In six of the schools there were available for use, or the pupils had access to, collections of mathematical instruments, models, and devices for mathematical instruction. The most notable of such collections is that of the Teachers College of Columbia University, New York City.

AIMS OF TEACHING.

In their mathematical teaching, all the schools aim at culture, as not being inconsistent with the special purposes which the school was designed to serve. Nine of the schools were either wholly or in part preparatory for college and three fitted for technical or normal schools.

In the six occupations listed under "vocational aims" one school gave preparation for the occupation of "draftsman;" two for that of computer; and five for that of accountant or merchant. There were no girls' schools that gave training for the professions of surveyor or civil or mechanical engineering. In addition to the aims classified three schools directed their mathematical training toward the equipment of the teacher of mathematics.

SUBCOMMITTEE 3. COEDUCATIONAL HIGH SCHOOLS IN THE EAST.

The questionnaire sent out by the Bureau of Education, at Washington, for this committee, met with a most generous response. The teachers of the secondary schools in the Northern Atlantic States—including Maryland—cooperated most heartily, and several hundred replies were received by the chairman. So many and so varied were these that a serious difficulty arose concerning the collating of the information there recorded. The district assigned to the committee was divided into sections, and the replies from a section given to each member of the committee for investigation. The committee worked for many weeks compiling a statistical report which was sent by the chairman to the chairman of the general committee on public general secondary schools, to be incorporated in the final report. To make a statistical report is fairly easy, but when it comes to giving a summary in essay form, it is almost impossible to do the subject justice. The fact that the answers to the questionnaire have been so many and so varied, and given by so many different individuals, who, of necessity, often have considered the questions from

a narrow point of view, makes it unwise to attempt to collate this large mass of material in a scientific way. While the reports of all the members of this committee were excellent, the one by Miss Mary Gould, of the Roxbury (Mass.) High School, covers so well the conditions existing in all the district, that the chairman has incorporated it below in this report.

REPORT OF MISS MARY GOULD, OF ROXBURY (MASS.) HIGH SCHOOL.

The following report is based on data sent by teachers of mathematics in the general secondary schools in the district assigned to the committee. As the number of pupils in schools sending answers to questionnaires varies from 32 to 2,380, and the conditions differ widely, we shall attempt to give only the consensus of opinion expressed regarding:

a. The organization,
b. The mathematical curriculum,
c. Examinations,
d. Methods of teaching,
e. Aims of mathematical teaching.

The mathematics department in schools where there are several teachers of the subject is usually organized under a head, who is responsible to the principal. While the committee, superintendent, and principal have full power to regulate the instructions of all schools in the vicinity, the curriculum and methods of instruction are seldom changed except at the request of those actually engaged in the work.

In the lowest class, committees of teachers selected by the head of the department set the limit to be reached by all divisions of the class from week to week, prepare uniform tests, state which topics are to be omitted and which are to be elaborated more or less fully than the text. Within these limits, each teacher has freedom of methods. In the second and third classes the same outline is followed but with greater freedom. In the highest class the work is usually in the hands of one teacher and there is little or no restriction.

On the teaching staff of the department there is usually at least one who teaches only mathematics, but there are others who teach other subjects also, often science, sometimes English, foreign languages, or history. In the larger schools little or no outside work in the supervision of athletics, journalism, or pupils' organizations is expected of a teacher, although in most cases they have the work which a session room entails.

Most cities and towns in this region have but one secondary school, so that contact with other teachers and cooperation with them comes mainly through mathematical associations. By discussions carried on at the meetings of such associations the college indirectly influences the methods of presenting mathematics at the secondary schools. We regret that the secondary schools do not in their turn help the elementary teacher.

Boys and girls are sent to a secondary school after an eight or nine year course in the grades, at about 15 years of age, to take a four-years' course (some schools give a three-years' commercial course). They are usually obliged to take algebra and geometry in the first and the second years respectively; college students are required to continue a third, and in some cases a fourth, year study of mathematics. Promotion comes annually by class, about 10 to 15 per cent graduating each year.

The curriculum of the mathematics department is largely determined by the object which the school has in view; in the college preparatory courses the college admission requirements mold the course; in the general and commercial courses, the school board and principal prescribe such studies and textbooks as seem to lead toward the ultimate purpose of the pupil. Teachers may, on the recommendation of the principal, modify the requirements by application to the school board. Although correlation of the different branches is not highly developed, there is a tendency to make

mathematics a progressive subject, and to correlate it with physics, chemistry, and mechanical drawing.

In running through the curriculum of this department, we shall omit an outline of arithmetic as it is taught first in the grammar grades.

In algebra the order of the topics is:

First year work.

Fundamental notions and operations, removing and insertion of parenthesis, simple equations without fractions; symbolic expressions preparatory to problems, solution of problems leading to equations of first degree without fractions; factoring (usual cases), highest common factor, including cases which can not be done by factoring; lowest common multiple, including some unfactorable cases; reduction, addition, subtraction, multiplication, and division of fractions, complex fractions of medium difficulty, solution of numerical and literal equations of the first degree containing fractions, problems leading to such equations; simultaneous equations in two and three unknowns (three usual methods of elimination), both numerical and literal, problems leading to simultaneous equations. Powers and roots of monomials, squares, cubes, and a few higher powers of binomials, square root of polynomials and of numbers. Theory of exponents, simple cases, without formal proof of the laws of exponents, surds, addition, subtraction, multiplication, and division of surds, omitting complicated cases; harder problems than before leading to equations of the first degree. Pure and affected quadratics, completion of the square and solution by factoring. Numerical quadratic equations and some simple literal ones. Some of the simpler cases of simultaneous quadratic equations. Problems leading to all kinds of quadratics studied. Graphs and graphical methods are used to some extent. The aim in this course is skill in manipulation, and not primarily to develop logical power.

Geometry, second year work.

The usual amount of plane geometry, theorems, constructions, and computations. In the prescribed course, only the simplest originals, theorems, and constructions are required, but a considerable number of numerical examples based upon theorems proved are required. The subjects of loci, incommensurables, and variables are very briefly handled.

In the elective (college preparatory) course a much larger number of originals (500 to 600), both theorems and constructions, is required. Loci, incommensurables, and variables are required and are fully treated. In this year four periods weekly of prepared lessons and one unprepared lesson are given.

In third school year, plane geometry is reviewed for half the year by college preparatory students, and greater attention is paid to originals, loci, variables. The usual amount of solid geometry (including elementary spherics) with exercises is followed for half a year, by those preparing for scientific or technical schools.

The course in geometry just described is the one usually followed in the larger schools, but the committee is not willing to say that it is the universal course; in most schools, however, geometry is taught either for two whole years, or for parts of two years. The periods per week are usually three hours if the two whole years are taken in preparation. It is very hard to get a course which would even approximate a universal one in this subject, because the methods and the time of preparation vary exceedingly in the different schools.

In the third year all college preparatory students have one-half year of algebra, five periods a week. The work of the first-year algebra is reviewed, but the harder examples omitted on first reading are mostly done, and the following new subjects are taken up: Theory of exponents and surds completed. The demonstrations of the laws of operation now required. Interpretation of negative, infinite, and zero results and simple indeterminate forms, imaginaries, harder literal quadratics and simultaneous quadratics (all the principal methods of solution), theory of quadratics, remainder and factor theorem, ratio, proportion, progressions, binominal theorem for positive integers (proof by mathematical induction required), simple inequalities.

In fourth year for half the year the following subjects are taken: Permutations and combinations, method of indeterminate coefficients, resolving into partial fractions, binominal theorem for negative and fractional exponents, plotting of equations and graphs, and theory of equations (the usual amount), including transformation, Sturm's theorem, and Horner's method of solving numerical equations of higher degrees.

In fourth year (second half) the usual amount of plane trigonometry, including solution of trigonometrical equations, solution of right and oblique triangles, applications to concrete problems, use of logarithms and tables.

Teachers agree that examinations produce uniformity in work and prevent marked divergence in the instruction. Tests are required for admission, and for graduation as well as during the course.

In development of the subject a teacher is allowed to use any method that brings results; at times teachers use the laboratory, heuristic, or individual methods; they find museums, colored crayons, measurement, and computation, paper folding, models, and cross-section paper helpful. To some extent algebra is combined with geometry; solid with plane geometry; and geometry with trigonometry.

Observational geometry should precede deductive geometry, but seldom does; and outdoor work in mathematics is decidedly in its infancy.

After the pupil has attained some degree of skill in manipulation from his first year's work in algebra, emphasis is placed on the logical relations found in deductive geometry and increasingly thereafter in his continuation of algebra. The teacher judges of the pupil's progress by his oral recitations, written home work, written class work, and voluntary original work. A greater per cent fail than in any other subject, estimates from 10 to 25 per cent being sent in. While it is desirable to estimate separately the pupil's measure of proficiency in numerical accuracy, algebraic manipulation, knowledge of geometric facts, comprehension of successive logical dependence, and mathematical invention and ingenuity, no system of marking has yet been devised recording such estimates.

Most schools have a hierarchy of aims in their mathematics course; the instruction strives to promote general culture, to prepare for college, or technical school, or if the pupil is to go out into the world immediately, to suggest the occupation of homemaker, civil or mechanical engineer, surveyor, draftsman, computer, merchant, or skilled worker in various local manufactures.

Two topics have been considered by the committee, which were not on the questionnaire: (1) the advisability of separating the girls and boys in the mathematics classes; and (2) the ability of the girls as compared to that of the boys.

There seems to be no general movement to make such a segregation of sexes in mathematical classes, although it is almost unanimously believed that such a step would be advisable in certain grades of the work. In the schools where a large number of boys are preparing to enter the higher technical schools, the classes in higher mathematics are composed entirely of boys, and much better work is being done than in the corresponding mixed classes. Even in the elementary subjects it has been found in such schools that better results are obtained if the segregation takes place early in the course. However, a strong objection to this scheme has been made by some members of the committee. It seems to be the general opinion that the average boy shows more ability in mathematics than the average girl, but also that he does not work so faithfully. In a mixed class this greater ability of the boy, and the greater faithfulness of the girl, react most advantageously on each other. This question of the difference in ability of the girl and the boy is a many-sided one. Every teacher in a mixed school has found in the mathematics class many girls who could do the work as well as the boy, and who by steady application to work has obtained a higher mark than he. But the average girl meets the situation at the start with a prejudice "ex matre," and is

therefore handicapped. Another fact that is noted is this: In elementary algebra the girl does as good work as the boy; in geometry not nearly so good. In advanced algebra and trigonometry, the boy shows an ability which is far ahead of the girl. Yet, we are free to confess that his greater natural ability is often outclassed by the steady, patient endeavor of the girl Perhaps it would be safer to say that the girl does not show as great an ability as the boy, even though she may have it.

The committee finds from the replies which were sent in that the conditions are still somewhat chaotic in many schools—a lack of organization; crowded curricula; the teaching of algebra and geometry by many teachers who have neither liking for the subjects nor ability to teach them; the use of poorly prepared text books; and usually the use of methods which are mechanical and ill-adapted to the needs of the present generation of secondary school students. An honest effort is being made in many schools to reform these conditions, and the committee sees signs of a coming era of saner methods, of better departmental work, and of closer correlation of subjects pertaining to mathematics.

SUBCOMMITTEE 4. COEDUCATIONAL HIGH SCHOOLS IN THE MIDDLE WEST.

INTRODUCTION.

Source of data.—In preparing this report the committee has secured data from a number of sources. Reports of various mathematical associations and of boards of education and magazine articles have furnished a great deal of material. Some of the information has been secured by correspondence. The main source of information, however, has been the questionnaire sent out by the United States Bureau of Education. This questionnaire was sent to every high school in the Middle West.[1] About 600 replies were received by the chairman. These replies were sent to each member of the committee, who examined them carefully for data on one or more points of the report. The results of this examination, together with matter secured from other sources, are presented in the following statement.

ORGANIZATION.

Departments.—Except in the high schools of large cities the mathematics can not be said to be organized into departments. Like the other subjects, it receives the attention made possible by the size of the teaching force. If this teaching force is small, mathematics is taught by those who have other subjects also. In the large high

[1] The "Middle West" was regarded as including the States of Ohio, Indiana, Michigan, Illinois, Wisconsin, Missouri, Iowa, Minnesota, Kansas, Nebraska, North Dakota, and South Dakota.

schools a better organization is found. One or more teachers give their entire time and energy to the teaching of mathematics, and thus the "department" of mathematics is formed.

About one-fifth of the replies to the questionnaires state that the schools have the work in mathematics organized into departments. The department heads are appointed in some cases by the board of education, in others by the superintendent or principal. The position is sometimes held by the right of seniority. No special rule determines the method of appointment; it usually depends on the wish of those interested.

The head of the department.—The duties of the head of the department vary. In some schools he has charge of the organization of the mathematics courses and must see that the teachers in the department effectively present the different subjects. He calls meetings of the teachers of the department, presides therein, and, subject to the control of the superintendent and principal, he is in a general way responsible for the work of the department.

In other schools he is considered a responsible person on whom the superintendent or principal depends to carry out his general policy or some detail which concerns the department as a whole, but he is not responsible for the organization and has nothing to do with the class work of other teachers, the superintendent or principal keeping direct control of that phase of the work. The greater number of organizations are of this type.

Teaching other subjects.—Three teachers of mathematics out of every five teach other subjects also. About 20 per cent have to do with athletics and one-half that number are required to do clerical work. In many of the smaller schools the mathematics is taught by the superintendent, who must divide his energies between class work and school supervision.

Transfer of pupils.—Chicago, Cleveland, Cincinnati, and Indianapolis are the principal places reporting the organization of the subjects to be such as to require the pupils to be transferred from one school district to another. Cleveland, for instance, has a number of high schools. These high schools do not offer the same courses. A pupil in one district, wishing to take a certain line of work, may have to be transferred to the high school of another district, as the one in his neighborhood may not offer the desired course. On the other hand, St. Louis offers the same courses in each high school and, consequently, there is no reason for transfers. This is true of most cities having more than one high school.

Relation between mathematics in the high school and in the grades.—High-school mathematics is most closely articulated with the mathematics of the grades in the smaller schools. In these places, especially if the superintendent has charge of the mathematics classes,

the connection is close and the bearing of one on the other is marked. In the larger schools the connection is not so noticeable. While the course of study is arranged in sequence, and promotions are made on basis of the pupil's ability to do the high-school work, there is nothing more. The high-school teacher of mathematics has nothing to do with the instruction of that subject in the grades. With some exceptions, he knows little or nothing of the pupil until he is enrolled in his class. About 50 (mostly superintendents teaching mathematics classes) report that they have direct influence on the grade work. About 100 state that their influence is indirect; the remainder, that they have no influence.

Determination of what is to be taught.—With few exceptions the university requirements regulate the mathematics taught in the high school. In some States the State board regulates, while in others the State superintendent has such authority. Naturally the relations between the university and the high school are not as close as between the high school and the grades. The university generally states the minimum amount of work of good quality which will secure admission for the high-school graduate.

Relations of the universities and the high schools.—The leading universities have at least one conference a year with the representatives of their affiliated schools. At these conferences mathematics sections discuss subjects from the standpoint of the university and from the standpoint of the high school. In many cases joint committees are appointed to make reports. In this way the two institutions understand each other better than formerly. As a consequence there has been a helpful modification on the part of each. The high school realizes more and more that it, as well as the university, is entitled to an effective part in the decision of the common questions of school life. At one time the university expected the high school to adapt itself to university requirements; now the leading universities do not hesitate to adapt themselves to the needs of the high school.

Associations of teachers of mathematics.—There are three kinds of associations of teachers of mathematics—the sections in the conferences of affiliated schools, the sections in the State associations of teachers, and the associations independent of other organizations. The first has been described, the second is similar to it; it will be sufficient to mention one of the third.

The Central Association of Science and Mathematics Teachers is the main organization that has influenced the teaching of mathematics in the Middle West. Its members are found in all the States covered by this report. It began under its present name in 1902. Specially appointed committees have prepared reports on the different topics in secondary mathematics, and those reports have been widely

distributed. They have had a marked influence on the high-school courses. Reference to the work of these committees is made in another part of this report.

The above influences have been partly the cause and partly the result of the activity on the part of the teachers in the Middle West in breaking away from the traditional lines in mathematics teaching. This activity is more or less manifest in all parts of the United States, but probably in no place as much so as in this district.

Miscellaneous statistics.—The high-school pupil enters at 14 years of age. He has had eight years in the grades, which may have been preceded by another in the kindergarten. Four years in the high school are necessary for graduation.

Twenty-five per cent of the reports state that four years of mathematics are offered, 40 per cent three years, and the remainder from one to two and one-half years. Where the schools are large enough there are different courses, some of which may require four years, others only one.

The percentage of the pupils entering that grade varies greatly. The minimum is 1.5 per cent; the maximum 62. The greater part is from 12 to 25 per cent.

No racial restriction of pupils is made except in a few places near Mason and Dixon's line.[1]

In 55 per cent of the schools annual promotions are reported; in 45 per cent semiannual promotions; 25 per cent report promotion by classes; 75 per cent promotion by subjects. In nearly every school mathematics is required in the first and second years; in the third year a number offer electives; 118 report no mathematics in the fourth year; 44 report that other mathematical subjects, as commercial arithmetic, mathematics for artisans, etc., are required.

THE MATHEMATICAL CURRICULUM.

Determination of the curriculum.—As a matter of fact, the university is the most influential factor in the determination of the curriculum. Each university or college sets up its own entrance requirements and admits graduates from high schools whose work meets these requirements, and the high school that wishes to stand well tries to conform to the standard so set up. The main outlines of the curriculum being thus virtually fixed, the superintendent, principal, or head of the department arranges the details of the course, and, finally, the school board must give its stamp of approval.

Various modifications may be made, such as the introduction of mathematics suited to a commercial course or to manual training. Such modifications are usually introduced by the school authorities

[1] The parallel of latitude 39° 43′ N., famous in history as the boundary line between the former slaveholding States and the others.

acting upon the recommendation of members of the department of mathematics.

The subjects taught.—In general, the courses consist of arithmetic, algebra, and geometry, and a few schools offer trigonometry and advanced algebra. In most places these branches are not correlated or taught simultaneously, but the prescribed amount of work in one branch is completed before another is studied. Correlated work is, however, to be found in many places. Some of the schools in the Middle West have been especially aggressive in this respect. Reference will be made to them later in the report. Attempts have also been made to correlate with physics. Manual-training schools have introduced courses in mathematics, which it is believed will be more helpful to their pupils than those usually prescribed, and commercial schools are making the same experiment.

Promotion of pupils.—Nearly all the high schools base the work of any one term on the satisfactory completion of the prescribed work of the preceding term. If his work in mathematics is satisfactory, the pupil is permitted to take the next term regardless of his success in other subjects. This marks a decided advance over the condition which existed a few years ago, when the pupil was compelled to repeat his work in all subjects, if he failed in one. As a rule admission to other subjects does not depend on mathematics in any case except physics, chemistry, mechanical drawing, and machine-shop work.

The scope of the courses.—The courses in algebra, geometry, trigonometry, and arithmetic are usually in close conformity with the current textbooks. There are, however, some variations from this, since in most cases teachers are at liberty to use discretion as to the order and emphasis of topics. As has been stated, attempts have been made to break away from the traditional order of things, and these attempts have furnished many topics for discussion before the mathematical organizations. These discussions, in turn, have resulted in some modifications, but it is too soon to make definite statements as to the outcome. It is sufficient for the purposes of this report to mention the existence of such a movement and to give in the sequel its main characteristics.

(a) Algebra.

Scope of the customary course.—Most of the high schools give a course in algebra such as is presented in the well-known current textbooks. The details of this course need not be discussed here, but it will be sufficient to refer the reader desiring further information to these textbooks.

Modifications proposed.—The Middle West has been especially active during the last three years in discussing modifications of the algebra courses. A number of reports have been made and the fol-

lowing statement is intended to be general enough to include their main points:

The work in high-school algebra should consist of an elementary course given during the first year and a more advanced course given not earlier than the third year, after demonstrational geometry. The first year should train pupils in the solution of problems by means of the equation, rather than exercise them in abstract manipulation. The later course, covering at least one half year, should include the demonstrational work and all topics usually given in elementary algebra. The kind of manipulation necessary for advanced work in mathematics is best emphasized in this course.

Proposed omissions.—In consequence of the central idea proposed for the first year, certain of the topics usually given should be omitted and the order of those retained should be changed. It has been suggested that the following topics be omitted in the first year:

Complicated brackets.
H. C. D. and L. C. M. by division.
Remainder theorem.
Complicated complex fractions.
Simultaneous equations in more than three unknowns.
Binomial theorem.
Cube root of polynomials.
Formal study of the theory of exponents.
Extended study and manipulation of radicals and imaginaries.
Equations containing complicated radicals.
Simultaneous quadratics except one linear and one quadratic.
Theory of quadratics.

A proposed order of topics.—The following order of development of topics has been suggested:

Additional problems; equations; abstract addition.
Subtraction; transposition in equations; numerical equations; simultaneous equations (simple); elimination by addition and subtraction; abstract subtraction; simple brackets.
Multiplication and division taught together; complicated problems in long division may be deferred till later; equations involving multiplication; easy fractions and fractional equations (monomial denominators); easy involutions; simultaneous equations (other methods of elimination).
Special cases of multiplication with easy factoring.
Solution of quadratics by factoring.
Roots and radicals.
Pure quadratics; affected quadratics by completing the square; simultaneous equations involving one quadratic and one linear.
Multiples and common divisors by the factoring method; addition and subtraction of fractions.
Multiplication and division of fractions; easy complex fractions.
Harder fractional equations; verification of roots.

Character of the problems.—Problems should have a concrete setting; they should be valuable and come within the scope of the pupil's experience; they should be abundant and, as far as possible, be introduced with each topic studied; they should be classified according to subject matter, and most carefully selected and graded. It is particu-

larly necessary that such problems be carefully graded beginning with the simplest, and that elementary conditions be comprehended.

The second course.—The later course should look upon algebra as a study of functions, and the various topics should be treated from this point of view. This course prepares for college, and need not be taken by pupils who plan to stop study after graduation from the high school.[1]

(b) Geometry.

Scope of the customary course.—Most high schools give a course in geometry such as is found in the well-known current textbooks, to which the reader is referred for details of the course.

Modifications proposed.—The modifications proposed in geometry have not been as extensive or as radical as those proposed in algebra. A study of the various reports shows that the following points have been considered and strongly recommended.

The introduction should be a course in construction. This is to familiarize the pupil with geometrical ideas and, for the time being, no attempt should be made to develop the formal logical processes.

"A free use of assumptions is recommended, yet it is essential that all propositions used explicitly in a formal demonstration be recognized as either previously proved or as belonging to the list deliberately left unproved." Rigorous proofs should not be demanded at the beginning. This does not mean that the demonstration should be a loose one; but rather that the exactness which is usually required in the beginning shall be recognized as a matter of growth and a result of later work.

Such propositions as "All straight angles are congruent," "All right angles are equal," "Circles having equal radii are congruent," should not be demonstrated, as is usually attempted at the beginning of a course. It is sufficient to regard them as direct inferences.

Propositions should be omitted which are obvious, too difficult, or unusual. List of omissions: Square of side opposite acute angle, square of side opposite obtuse angle, division in extreme and mean ratio, inscribed decagon, symmetry, theorems on limits, incommensurable cases, maxima and minima, sum of squares of two sides equal to twice the square of half the third side increased, etc., difference of the squares of two sides, etc., in any triangle the product of two sides equals the diameter of the circum-circle multiplied by, etc.

These omissions give opportunity for emphasizing such topics as "congruence of triangles, similarity of triangles, Pythagorean theorem, properties of circles, mensuration theorems concerning plane figures, properties of spheres."[2]

Solid geometry.—The work in solid geometry immediately follows that in plane geometry in many schools. In others a course in

[1] The above statement on algebra is taken almost entirely from the "Proceedings of the Eighth Meeting of the Central Association of Science and Mathematics Teachers." The committee could have quoted almost the same statements from other reports. Special mention should be made of the Illinois report, the Indiana report, and the reports from Missouri and Wisconsin.

[2] The quotations are from the Illinois report on geometry, 1911.

algebra is given between plane and solid geometry. In either case more emphasis is laid upon the formal deductive side than in the first year. Many teachers develop the notion of functionality by the introduction of trigonometric functions.

(c) Mathematics.

Character of the courses.—Some of the schools report courses in "mathematics." This differs from the work usually given, in that arithmetic, algebra, geometry, and trigonometry are unified. The traditional barriers which have kept these subjects separated have been broken down. Other subjects, notably physics and chemistry, are made a part of this unification. The old cut and dried forms of treatment are ignored. As one report states, "If we know arithmetic, algebra, geometry, and trigonometry in their relations to one another, there will be no hesitation about introducing one subject into the teaching of another wherever such material is needed."

The high schools at Lincoln, Nebr., Indianapolis, Ind., and Cleveland, Ohio, have been especially prominent in this line of work, the first named being a pioneer. Small schools where it is necessary to combine such departments as mathematics and physics, or to have all the mathematics taught by one teacher, are among the number reporting this plan of work. There is no uniform arrangement of subject matter in the schools teaching "mathematics." The following statement, however, is representative:

> The topics usually considered in the courses in algebra and geometry are distributed throughout the entire course on the spiral plan, the more simple and concrete ideas at first; the more complicated, difficult, and abstract ideas later. Algebra predominates in the first year, geometry in the second. Arithmetical work is carried on throughout the entire course in conjunction with other topics wherever it naturally enters. Some of the trigonometric functions may be made use of in connection with the triangle.[1]

EXAMINATIONS.

Examinations for entrance to the high school.—The first year of the high school is regarded as the ninth grade of the local system of graded schools. A pupil passing the eighth grade can enter the ninth just as one passing the seventh can enter the eighth. But it is different with pupils entering from another system of schools. In that case an entrance examination is required. If the pupil is a candidate for the first year of the high school, this examination must be such as to satisfy the school authorities that he has a knowledge of the eighth-grade work; if he is a candidate for advanced standing, he must show by examination that he is worthy of advanced credit. Occasionally schools take candidates on trial. A few weeks' test in the classroom

[1] The quotations are from a report entitled "Unifying Mathematics," in the "Proceedings of the Eighth Meeting of the Central Association of Science and Mathematics Teachers."

will be regarded in the light of an examination. Arithmetic is one of the principal subjects in the entrance examination.

Examinations for entrance to college.—Universities do not give entrance examinations to graduates of their own accredited high schools. But if the candidate for entrance is a graduate of a high school not on the accredited list, an examination is required. The North Central Association of Colleges and Secondary Schools has brought it about that graduates from the strong high schools of this district can enter without examination any of the higher institutions of the Middle West.

Effect of examinations.—We pass next to the consideration of the following question from the questionnaire: "What is the effect of these several classes of examinations (*a*) on the teaching of mathematics in your school? (*b*) on the pupil's attitude toward the subject?"

The replies are tabulated as follows: To the first, 249 say the effect is good; 24 that it is bad; 54 say it is not noticed. To the second, 194 say the effect is good; 34 that it is bad; and [illegible]3 say that it is not noticed. It is interesting to note from these replies that more see good effects on teaching than see good effects on the pupil's attitude.

METHODS.

The methods in use.—The replies to the question concerning the methods used in teaching secondary mathematics ~~do not~~ always make distinction between "methods" and "devices." Those used most are the laboratory method, individual method, measurement and computation, and cross-section paper. The largest number state that they use measurement and computation. Some of the replies state that all these methods and devices were used during the year.

As already stated, a number of schools make a combination of arithmetic, algebra, geometry, and trigonometry. It is not to be inferred, however, that this combination always means correlation. It may mean the teaching of the subjects simultaneously—for example, geometry three hours in the week and algebra two.

The following special methods or schoolroom devices are taken from the reports:

1. Individual problems.
2. Have the pupils make models.
3. Find the value of π by rolling a disk on a yardstick.
4. Use colored crayons.
5. Small drawing board, T square, and triangle for each pupil.
6. Use a "composite figure" from which the pupil selects material to prove various propositions.
7. Make lantern slides from construction problems thoroughly worked out.

8. Use stereopticon.
9. Have small sections in the class.
10. Give geometrical drawings first.
11. Complete reviews should be printed on the mimeograph and distributed to each pupil.
12. Pupils should make figures in solid geometry from prepared clay and toothpicks.
13. Use equation balances.
14. Use the transit.
15. In Book VI use a large sheet of cork and knitting needles.

When logic should be emphasized.—Of the replies to the question: "At what stage in the pupil's progress should the emphasis be placed upon his grasp of logical relations rather than upon his manipulative skill?" 40 per cent state that it should be during the second year, 25 per cent the first year, 20 per cent the third year, and the remainder during the fourth year or "throughout the course." These data support the claim of teachers who believe that geometry should be given during the second year rather than the first. As has been stated, this subject is found in most of the schools after one year of algebra.

Grading.—Pupils are graded on oral recitations, written home work, written class work, voluntary class work, written examinations at stated intervals or at the end of the term, and on the teacher's general estimate of the pupil's power and achievement. Oral recitations are used by nearly everyone, as is shown by the fact that 550 stated they always considered them. Examinations, written class work, and the teacher's general estimate are favored in the order given.

Exemptions.—In many schools exemptions from examination are given; that is, if a pupil attains a specified grade, say 90, on his class work, home work, etc., he need not take the final examination.

Failures.—The percentage of pupils who fail runs from 10 to 85 per cent. No other point considered in this report presents such extremes. The greatest number of replies state that 10 per cent fail. Twenty per cent, 25 per cent, and 15 per cent occur in the order given. The replies giving 10 per cent of failures are usually from the smaller schools where the number is such that the individual pupil can be looked after more closely and the pace adapted to his ability. The large, well-organized schools, where the standards are highest, report not less than 20 per cent, the number usually being between 25 and 35 per cent.

Discriminating marks.—The questionnaire asks, "Are the methods of marking mathematics in your school so arranged that it is possible to conclude from the pupil's marks on record whether he is proficient in numerical accuracy, algebraic manipulation, knowledge of geometrical facts, comprehension of successive logical dependence, and

mathematical invention and ingenuity?" Eighty per cent report "No" and 20 per cent "Yes," though 75 per cent say such a record is desirable. Some state they include all of these points, though more emphasis is placed on one or two than on the others.

Various suggestions were made for securing such a method of marking:

1. Use of letters—
   Ae, excellent in accuracy.
   Mf, fair in manipulation.
2. Card system.
3. Grade 20 per cent as perfect in each.

Mathematical clubs.—Some of the large schools have mathematical clubs. The Engineering Club at the McKinley High School, St. Louis, Mo., is typical. Its membership is limited to 35, and restricted to pupils of the junior or the senior class, whose general record is good and whose interest in mathematics is such as to cause them to seek membership. One of the mathematics faculty is moderator and sees that the efforts of the members are directed along worthy lines. Committees arrange the programs and see that each one does his part. It is understood that the success or failure of the semester's work depends on the club and not on the faculty representative. So far the results have been very satisfactory.

The following are some of the topics considered during one year:

Field work—

Surveys and computations necessary for a proposed extension of a street railway line, including maps and profiles.
Surveys for a proposed extension of a city street.
Triangulation, or measurement of inaccessible distances.

Reports—

Principles of bridge building.
Gold mining.
Ice-making plant.
Electrical transmission.
Sewage disposal.
Raising of the *Maine*.
Points dealing with the construction of the new bridge, sinking of the new caissons, etc.
Efficiency of electric batteries.
Problems dealing with the aeroplane.

AIMS OF MATHEMATICS TEACHING.

Character of the replies.—The information which the committee secured from the questionnaire on the "aims of mathematics teaching" is not satisfactory. The statements were in many cases indefinite. In view of this fact it seems advisable to tabulate the replies and to make no attempt at generalizations.

The general aim.—In answer to the question "Is the aim of mathematical instruction in your school general culture?" 400 said "Yes." Three hundred and fifty state their aim is to prepare for college; 125 prepared for technical school, while 25 do not make such a preparation.

Preparation for specific occupations.—The replies to the question concerning preparation for certain specified occupations are as follows:

1. Civil engineer	47 yes, 29 no.
2. Surveyor	35 yes, 36 no.
3. Draftsman	31 yes, 37 no.
4. Mechanical engineer	47 yes, 36 no.
5. Computer	45 yes, 38 no.
6. Accountant or merchant	171 yes, 31 no.

School teaching, agriculture, housekeeping, office work, carpentering, all receive frequent mention in the answers to this question.

SUBCOMMITTEE 5. MATHEMATICS IN THE COEDUCATIONAL HIGH SCHOOLS IN THE SOUTH.

Throughout the South, as in other parts of the United States, the public high schools that are not especially designed for vocational ends, as few are, are intended to follow an elementary school course of seven or eight years. Some of the States specifically prescribe courses of study necessary for approval by the State inspector. This approval in many cases is a prerequisite for the allotment of pecuniary aid from the State treasury, or for admission to the State university or to the State normal schools.

For the purposes of this investigation a list of questions was sent to school officials and high-school principals in all parts of this region. Replies were received from 80 of the persons so addressed. So far as those replies are pertinent, the following conclusions appear.[1]

ORGANIZATION.

In 61 per cent of the schools the principal has effective influence over the teaching of mathematics. In 47 per cent of the schools there is a head of the mathematics department or a special teacher of mathematics, selected in three-fourths of the cases by the school board or by the superintendent of schools, and in the other fourth by the principal. The teachers of mathematics have other subjects to teach in 59 per cent of the cases, have charge of or supervision of school athletics, journalism, or social activities in 24 per cent, and have clerical work in addition to teaching in 17 per cent.

The school is represented in associations of teachers of mathematics in 44 per cent of those replying; the effect of such associa-

[1] Percentages are of the number of schools replying to the question. Some of the questions were regrettably so framed as not to elicit full replies.

tions upon the teaching in the school is in a comparatively small proportion of cases reported as beneficial.

Throughout this region separate schools are provided for whites and for negroes. In some cases the statement is squarely made that separate schools with identical courses of study are offered to the two races, but there is a clear implication that in many of the smaller communities at least the opportunities for secondary education are restricted to whites.

Promotions are annual in 76 per cent of the schools replying, semi-annual in the others; by subject in 44 per cent, by class in 56 per cent.

The percentage of the whole school membership graduating each year varies widely, as shown by the following table:

Number of schools reporting.	Per cent of pupils graduated.
9	0 to 5.
21	6 to 10.
23	11 to 15.
13	16 to 20.
2	20 or more.

THE MATHEMATICS CURRICULUM.

Course.—The course of mathematics in 87 per cent is determined by the admission requirements of higher institutions. Decisive action is in all cases taken by the school board; the principal is an influential adviser in 82 per cent and the head or special teacher of mathematics in 71 per cent.

The different branches of mathematics are correlated with each other in 70 per cent, and the successive years of mathematics are dependent on the previous years' work in 96 per cent. (The replies to the former question undoubtedly indicate a higher degree of correlation than actually exists among the different branches of the study. The meaning of the question does not seem to have been clear.)

Some correlation is found between mathematics and physics in 58 per cent, and success in the algebra course at least is insisted upon for admission to the study of physics in 32 per cent.

Content of the curriculum.—Half a year of arithmetic is given in 80 per cent, and an additional half year in a few others, say 3 per cent. Algebra is given for a year at least in all the schools, for a year and a half at least in 40 per cent, for two years in 21 per cent, and for two and a half years in 3 per cent. Arithmetic is given after algebra, either as review or as "higher arithmetic" in 14 per cent. "Higher arithmetic" as used here means evidently more involved problems and commercial applications of the same principles that are treated of in ordinary arithmetic, with the occasional addition of such topics as mensuration and square root. There is no indication of any consideration of the relative accuracy of data, of the number of sig-

nificant figures as indicating the degree of approximation, or of any use of logarithms or the slide rule as an adjunct of the arithmetic work so-called.

Plane geometry is given for one year in all the schools, for a year and a half in 9 per cent of them, and one school reports plane geometry for two years. Solid geometry is given for half a year at least in 75 per cent of the schools, and for an additional half year in 6 per cent. Plane trigonometry is taught in 58 per cent for half a year: one school reports surveying, three report spherical trigonometry, one reports analytical geometry.

In the reports handed in, physics is quite frequently listed as a mathematical subject.

EXAMINATIONS.

Examinations are set for admission to 69 per cent, for graduation in 86 per cent, for admission to higher institutions in 20 per cent. The greater part of the schools are accredited to certain universities after inspection by a State or university inspector, and have their graduates examined only for admission to distant institutions which do not accept high-school certificates. The custom is growing of accrediting elementary schools so that their graduates are admitted without examination to the high schools in their region. Examinations for promotion or graduation have quite generally been waived for pupils whose marks reach 90 per cent in their class work.

More than 80 per cent of the schools reporting one or more of these examinations as in use consider their effect on both teachers and pupils as beneficial. The 90 per cent exemption is enthusiastically commended and also unsparingly condemned.

METHODS OF TEACHING.

In the replies to the question whether the methods of teaching named in the following lists were in actual use or were educationally desirable, the first column gives the order of frequency, the second the order of preference:

1. Measurement and computation.	1. "Laboratory" method.
2. Use of models.	2. Measurement and computation.
3. "Laboratory" method.	3. Individual method.
4. Individual method.	4. Use of models.
5. Use of cross-section paper.	5. Use of cross-section paper.
6. Combination of algebra and geometry.	6. Outdoor work.
7. Paper folding.	7. Observational geometry to precede deductive geometry.
8. Outdoor work.	8. Combination of algebra and geometry.
9. Observational geometry to precede deductive geometry.	9. Heuristic method.
10. Heuristic method.	10. Paper folding.
11. Combination of solid and plane geometry.	11. Combination of solid and plane geometry.
12. Combination of geometry and trigonometry.	12. Combination of geometry and trigonometry.

There is a suspicion that the word "heuristic" was unfortunately chosen, and that the word "combination" would have been with advantage replaced by "interweaving" or "blending." On this account the relative numerical standing of these replies is open to question.

In spite of the fact that fewer replies were made to the question of desirability than to the question of use, the desirability seems to have been considered with care, and the replies to be independently significant.

There are occasional reports of details of method, such as that geometry is taught "with applications to algebra," that the work is made "as practical as the textbooks will permit," that the pupils make their own models with string and cardboard. Where the number of originals given in geometry is stated, it ranges from 100 to 500; and one teacher states that 70 to 90 per cent of the time in algebra is devoted to problems.

A school superintendent writes that he has long suspected that "much of the algebra game is not worth the candle."

As to the question at what stage in the pupil's progress in mathematics emphasis should be placed upon his grasp of logical relations rather than upon manipulative skill, the replies range from the sixth grade of the elementary school (11 to 12 years of age), where arithmetic is the only mathematics in sight, to the last year in the high school, with perhaps a slight preponderance on the second year in the high school, where plane geometry is begun. On the whole this question does not seem to have been carefully considered by our correspondents.

The pupils are graded on the following grounds, arranged in the order of frequency:

Oral recitations.
Written class work.
Written periodical tests or term examinations.
The teacher's general estimate of power and achievement.
Voluntary original work.
Written home work.

The question whether methods of marking were such that the teacher could infer from the pupil's marks on record whether he was proficient in the several qualities listed below was answered by 45 teachers, of whom 60 per cent said that such inference was not possible; the question whether it was desirable was answered by 61 teachers, of whom 60 per cent gave a decided affirmative. The qualities referred to were:

I. Numerical accuracy.
II. Algebraic manipulation.
III. Knowledge of geometrical facts.
IV. Comprehension of successive logical dependence.
V. Mathematical invention and ingenuity.

Devices suggested for such a method of marking were specially ruled marking sheets or class record books. The comment was usually made that such a method of marking would be too cumbrous.

Only 2 per cent of the reply sheets received reported that there were mathematical clubs to which the pupils were admitted; only 11 per cent that there were collections of mathematical instruments, models, curiosities, or devices for instruction, accessible to teachers or pupils; and these collections were obviously in most cases only the modest outfit of apparatus for instruction.

AIMS.

The aim of the mathematical course in practically all of these schools, as indicated by the content of the studies, is college preparation, although 15 per cent of the replies do not acknowledge that aim. The purpose universally acknowledged is "general culture." Half of the schools reply to the question whether preparation for technical schools is a purpose of their mathematical study, and of these 75 per cent answer affirmatively.

As to the question for what particular occupations the mathematical work of the schools may be considered preparatory, the following is the order of frequency:

Accountant or merchant.
Civil engineer.
Mechanical engineer.
Surveyor.
Draftsman.
Computer.
Farmer.
Teacher.

Other replies were "practical work," trades, "practical finishing school." Many of the schools furnish from their graduates teachers for the elementary schools of the same region.

SUBCOMMITTEE 6. COEDUCATIONAL HIGH SCHOOLS ON THE PACIFIC COAST.

This report is the outcome of an investigation carried on by the subcommittee, through a questionnaire sent out from the Bureau of Education at Washington, D. C., and a somewhat more personal appeal sent out by the subcommittee to the public secondary schools of California, Oregon, Washington, Idaho, Montana, Utah, Colorado, Arizona, and New Mexico.

The subcommittee feels that, although about 200 replies were received, much of the information obtained is not particularly reliable,

because of the evident carelessness with which many of the answers were made. This very fact, however, leads the committee to draw certain conclusions about the prevailing type of mathematical instruction in the secondary schools. Although many of these conclusions are not particularly flattering to the teaching profession, we trust that they may lead some to a better appreciation of the present conditions and spur them to more systematic work. This report will follow the general topics as set forth in the questionnaire.

ORGANIZATION.

The larger schools naturally have a much more complete organization than the smaller ones, which fact leads us to classify the schools into two groups—those with 150 or more pupils and those with less than 150. This classification will in general mean city schools and country schools respectively, although, in the case of some of the union schools, we may find some of the first class in country towns.

The larger schools have a mathematics department, usually with a department head who is appointed by the school board or superintendent at the recommendation of the principal. In no case is the organization very formal, although in the best schools frequent informal and regular formal meetings of the mathematical teachers are held.

In the vicinity of the large cities, notably San Francisco, Los Angeles, Denver, Portland, and Seattle, the associations of mathematical teachers have a considerable influence on the methods of teaching. This is not so much the direct influence of papers read at the meetings of these associations as the influence which the writing of such papers has on the writer to bring the problem of teaching before him in a clear-cut form. In the best schools the majority of the teachers feel this influence, either directly, or indirectly through listening to the papers. The associations have a beneficial influence also in bringing about a uniform interpretation of the State university requirements. These requirements are the strongest influence upon the content and order of the mathematical instruction, and frequently papers upon new "requirements" bring out discussion which leads to a better understanding of the aim of such changes. Indeed, the universities welcome such suggestions as the associations are willing to offer, and thus these discussions may lead to a better adjustment of the "requirements" to existing conditions.

There are many schools which disclaim any influence from the State university. Nevertheless nearly every school desires to be accredited at the university; and even if this is not accomplished, the entrance requirements practically control the content of the courses offered. In fact, a California State law requires the high schools to offer a course preparatory to the State university.

The influence of the secondary-school instruction upon grammar-school work is not very perceptible, although in some cities where the principal of the high school is also superintendent of schools he has an influence over the curriculum of both schools, which naturally tends toward unifying the aims of instruction. This influence is felt perhaps more through the selection of textbooks than in any other way. Since the textbooks, once selected, must remain for four years in California, and six years in Oregon, a great degree of care is taken in the selection. The influence of the principals in such selections is rather strong, because teachers are often changed during the period of use of a book, making it impracticable to rely on their recommendations.

It may be said that in practically all cases the high school principal exerts the strongest influence in the selection of books, although his selection is subject to the approval of the board. In Oregon, however, an "unbiased commission of marked ability" selects uniform books for the grammar and high schools, thus taking the whole matter out of the hands of those directly concerned with their use.

Most of the teachers, especially in the small schools where more than one teacher is required for mathematics, spend a part of their time teaching other branches, notably science. In fact the student at the university who looks forward to high-school teaching fits himself to handle some second subject. In a majority of schools, however, the teacher of mathematics is not required to participate in other school activities, but it may be said that many do take advantage of the opportunity to become supervisors of student athletics.

From the standpoint of the teacher, then, the organization of mathematical teaching in the secondary schools may be characterized as very indefinite. Although State laws and university requirements tend to bring before the instructors definite topics for discussion and thought, the reaction on the part of the great majority of the teachers is very meager. Even where mathematical associations make a few of their members enthusiastic, the rank and file of the teachers do not concern themselves seriously with real problems of teaching.

The very fact that there were few replies from the large number of questionnaires sent out makes it plain to us that many of the teachers do not concern themselves vitally with the advancement of their profession. It is, however, undoubtedly true that the details of the work required of the teacher are so near to the limit of endurance that little time or energy is left for serious thought about methods or aims.

The formal organization of departments and active interest on the part of mathematical teachers are necessarily greatly hampered by these conditions. The interest being also somewhat divided between mathematics and other subjects, and the responsibility of selecting

books being partially shifted to others, produce a passivity on the part of the great majority which does not argue well for rapid future progress in mathematical teaching.

Conditions could be improved by a decrease in the amount of routine work, such as correcting papers, and a decrease in the size of classes. An increase in salaries would also, probably, attract more active and efficient men to the profession and thus bring about a better spirit among the workers. Such changes can not, however, be expected immediately, and in the meanwhile the mathematical teachers' associations and State inspection of the schools for the purpose of accrediting at the State university are the remedial agencies to which we must look for the advancement of the profession.

Most of the schools on the Pacific coast are so organized that students enter the high school after eight years of elementary training at ages ranging from 13 to 15, with an average of about 14.5. The course is uniformly four years, with four full years of mathematics offered in most cases. Commercial arithmetic is offered in some schools but is usually taken only by pupils who have a business career in view. In most of the schools only two years of mathematics are required, i. e., elementary algebra and plane geometry. Those students, however, who are preparing for the engineering colleges are required to take four years.

Promotion in some of the larger schools is semiannual, but in the smaller ones it is annual. This promotion is uniformly by subject and not by class, a student being catalogued in that class with which he is taking the majority of his work. The number of pupils who graduate from the older schools is about 10 or 12 per cent of the membership of the entire school each year, though in some cases the percentage is as high as 15.

From the standpoint of the pupils the organization of the schools is decidedly uniform throughout the Pacific coast States. A four-year course, with mathematics available each year, follows eight years of elementary school work; and promotions are annual and by subject. This uniformity makes changes from one school to another an easy matter for the pupil, and barring the difference in quality of instruction a student need not trouble himself to go to the larger cities in order to be prepared for any kind of further study.

This uniformity of organization, although it is due largely to the definite requirements of the State university, is also the result of public appreciation of mathematical training as an effective means of preparation for life. It is, of course, true that the State laws, which require the State universities to admit without examinations such students as come from accredited high schools, are largely responsible for the effort on the part of each school to maintain the required standard. A significant fact, however, is that were this law a serious

detriment to many young people, the public has the power to bring about the desired change of policy, not only as a State, but as individual towns, the school committees in which regulate the policy of their high school.

We may conclude then that, although the universities seem to control the organization of the high schools, yet this control does not in reality lie with even the State universities, for both public sentiment and the existing demands of high-school pupils really dictate the terms which the universities shall offer.

THE CURRICULUM.

The mathematical curriculum is usually determined entirely by the State university requirements, although the school board officially determines the question for any given school, and of course has the authority to offer courses with no mathematics, provided it offers one course preparatory for the State university. Consultation with the superintendent and principal is the formal method of this determination, but it is the prevalent custom to accept the subjects as laid out by the State university. The order and method of teaching is, however, more within the control of the mathematics teachers, and can be changed usually by conference with the principal. The State universities do not presume to dictate the order of topics or even of subjects. It is, nevertheless, true that many teachers and principals are greatly influenced by the preference which the State university professors may have for certain texts. Thus, if it is known that the State university staff has used or recommended a text which presents the subject in a certain order, either that text or another with a similar order of topics is liable to be selected by many schools. Indeed the publishers use the approval of certain texts by the professors as a strong argument for their adoption in the high schools.

Some attempt is made in the larger schools to correlate the algebra and geometry by teaching both as a continuation of arithmetic, and making them come under one general topic, mathematics. Separate texts are, however, used, and it remains largely with the individual teachers to bring about any really vital correlation. Correlation with physics, chemistry, and mechanical drawing seems to be somewhat widespread, but the correlation seems to mean chiefly that algebra and plane geometry are required as preparation for these subjects, or that some problems from these branches are introduced into the algebra and geometry courses. Original exercises are strongly emphasized, in some cases as many as 500 being worked in the year. Some attempt is made to obtain problems from "out of doors," but it can hardly be said that there are many practical problems used.

We find then, that the mathematical curriculum is determined by State authority of some kind, and is usually only slightly controlled by individual schools. The interrelation of mathematical subjects is not generally brought out, although a few of the most progressive teachers are trying to vitalize the work by breaking down the barriers surrounding each chapter of the text, and allowing the class to move about in a more unrestricted field, which gives them some concept of the real uses and significance of mathematics. For the most part, correlation either among mathematical subjects or between them and other subjects, seems to have no very definite meaning to the majority of teachers. Two causes may be cited as responsible for this. First, the average teacher has not the broad knowledge of mathematics beyond his subject which would enable him to appreciate the real significance of the various topics as they arise. He is not, however, wholly to blame for this, because the present conditions in secondary education do not give him time or money with which to put himself in touch with this broader knowledge. Nor does the remuneration offered in most schools seem attractive to those who have spent their time and money in thoroughly preparing themselves by the more comprehensive study of the subject. Having therefore no clear-cut understanding of how the subjects might be correlated, they are at a loss to know exactly what is meant by their correlation. Another cause is that the textbook publishers and writers, recognizing the desire on the part of the rank and file to have a text which can be accurately followed page by page, have given us books in which each chapter opens and closes a topic with as little reference as possible to other topics. Such texts run themselves, when once set in motion, and call for little teaching on the part of the instructor.

Under these conditions we can hardly look for much correlation or for much deviation from the presented order of topics, except in the case of a few of the best schools where an exceptional teacher has become aware of the possibilities of the subject.

EXAMINATIONS.

The general tendency seems to be toward fewer examinations and toward more than one recitation period for each examination. In most of the schools, pupils are admitted from accredited grammar schools without examinations, and are graduated from the high school on the basis of monthly or bimonthly tests, rather than on a final examination covering the whole course in mathematics, or even a whole subject. The State universities and many other higher institutions accept these graduates from accredited high schools, and thus the whole system tends to minimize the importance of examinations. There is considerable difference of opinion on the part of the teachers in regard to the desirability of formal examina-

tions. Some believe that they tend to produce superficial work on the part of both teacher and pupil, because the real aim of mathematical teaching is obscured by the desire to pass the examination. On the other hand, a large number express themselves as in favor of examinations as a means of producing thoroughness, as well as a means of emphasizing the aim of teaching, and of calling to the attention of the teachers the defects in their methods. They also claim for the pupils a larger interest and greater respect for the subject when examinations are prominent.

The elimination of final examinations, and even monthly tests, by means of high standing in daily work, seems to be a universal recommendation with those who advocate the examination as an incentive to good work. This would indicate that they consider the examination not so much a good thing in itself, as a spur to good daily work. If the pupil could be induced to prepare himself thoroughly for the examination, and then be excused each time from taking it, the examination would have served its purpose for all. Since, however, this is impossible, from the nature of the case, there seems to be little doubt that some sort of examination is necessary, though it be an evil.

METHODS OF TEACHING.

The specific methods of teaching mathematics classified as laboratory, heuristic, paper folding, etc., seem to have rather vague definitions in the minds of most of the mathematical teachers. The answers indicated a general use of "individual method," "measurement and computation," and "cross-section paper." A majority reported "models" and "outdoor work," and Colorado reports the use of "charts" in the review of geometry.

There is a decided disapproval of the "laboratory method" of teaching geometry. Practically none of the schools on the Pacific coast use it, and many express the opinion that it leads to looseness on the part of both pupil and teacher. Observational geometry is also opposed upon the same ground. If the "laboratory method" and "observational geometry" were used as an introduction to the formal demonstrative geometry, and the mere observation of facts by the class were only a means of bringing more clearly before the pupils what the real conditions are, the method might serve a very valuable end. The prevailing opinion seems to be, however, that the work tends to degenerate into substituting the evidence obtained from the observation of concrete objects for the logical proof which should follow such an observation. In the hands of some instructors the habit of careful and accurate reasoning may not suffer, but may rather be benefited from this kind of work. To the majority, however, it is a strong temptation to laxness.

Concerning the time for emphasis on logical reasoning, opinion is somewhat divided between the second year and the first half of the

third. The middle of the second year seems to be the most generally accepted time. The fact that the second year is the time when geometry is begun in most of the schools leads us to question whether the opinions expressed mean that the second year is really the time or that, because geometry is taken at this point, it is therefore the best time to begin logical reasoning. The fact that logical reasoning upon algebraic theorems is somewhat more difficult than that on geometrical theorems makes it true that very little attempt is made to force logical reasoning into the first year of mathematics.

It may, however, be true that if high-school mathematics could be begun with some simple forms of geometrical reasoning we should get better results in all of the mathematical work. This process might eliminate from the pupil's mind the idea that all mathematical generalizations can be learned by heart, and thus avoid some of the bad failures in geometry. If we could start the pupil with the idea that he can and must reason out the steps in a great deal of his mathematical work, should we not have a better attitude toward those parts of the work in which reasoning is the only possible method? First impressions are so lasting that it seems desirable that those first impressions should be more in accord with the real spirit of mathematics than to have them so nearly in line with the pure memory processes of the grammar-school work. Memorizing rules for each section of algebra is certainly not conducive to conceiving of algebra as a generalization of arithmetic. Some topics now given in algebra would probably be crowded out by this process, which must of course be somewhat slower, but in the end the increased power to think might well compensate for such topics as algebraic cube root and the division of long polynomials. The demand by the colleges for the "notion of functionality" [1] in algebra is along this line of making algebra a more logical subject.

All classes of work—oral, written home work, written class work, original work, and the teacher's general estimate of the pupil's ability—come in for about two-thirds of the final mark, while the formal examination gets credit for about one-third.

The tendency toward fewer examinations signifies a decreasing confidence in the examination as a test of ability. This lack of confidence, of course, makes the teacher count the examinations as a smaller and smaller fraction of the total mark and advocate the high standard of daily work as a sufficient test for the final grade.

In regard to disintegrating the marks into numerical accuracy, knowledge of facts, mathematical invention and ingenuity, it may be said that there is no systematic attempt to make such a separation. It is true, however, that many teachers unconsciously have a more or less defined method of marking work on this basis. When

[1] See requirements of University of California.

brought squarely before us, the details of such disintegrated marking impress most of us as too complicated for the rapid work which we have to do, but with a special class record book and smaller classes such a system might give us better information than we now get about a pupil's ability. It seems probable, however, that, if we could have the small classes, the intimate acquaintance which we should then have with the pupils would be a much better basis for marking than the details of such a system.

There seems to be a somewhat general feeling that the best teachers adopt no particular method of teaching, but try to acquaint themselves with the underlying principles of the subject and with the general nature of the pupils before them, and then adopt that method which will accomplish the purpose of the moment with a reasonable degree of success. Although the answers seem to indicate vagueness, on the part of the teachers, in regard to exactly what method has been adopted, this is perhaps due to the fact that those teachers who have considered the question of method do not consider themselves bound to any specific one. This is probably as true of marking as of methods of presentation.

AIMS OF MATHEMATICS TEACHING.

General culture and preparation for technical schools and colleges are both acknowledged by most of the high schools as the aims of their teaching. There is a decided denial of vocational aims, such as civil engineering, surveying, accounting, etc., although some schools record all these as secondary aims. It is probably true that the teachers and principals feel rather strongly on this point on account of the continual demand by pupil and parent for studies which will lead directly to some vocation. The attitude of the teacher has become, rather naturally, that of restraint for those pupils who wish to take only such subjects, and even only such topics as will furnish knowledge in the direct line of a given vocation. The teachers realize, as perhaps parents and children never do, that a groundwork of fundamental principles and processes must be laid before any success can be attained in the chosen vocation, be it engineering or accounting. If we should ask the pupils why they study their mathematics, and could get a reply for which they had not been coached, we should probably find more emphasis placed upon the vocational than the cultural aim.

Although the aim does not always keep itself prominently before most of us during the details of everyday work, we are evidently more or less conscious of very definite aims in our teaching of mathematics. Were this not true, the attempts to cover certain topics within certain stated periods, and the change of texts for the purpose of a better presentation of what we think is desirable, would not concern us very deeply. These aims may in many cases be only

preparation for some college or technical school, but general culture has a prominent place in the minds of most of the thinking teachers.

On the whole the results of this investigation have given us an insight into the conditions of the Pacific coast schools which, although very disappointing in many respects, is sufficiently general to be of value in feeling the pulse of the whole country. Any opinion in regard to the best remedy for the present conditions is hazardous, but the committee feels justified in calling attention to the fact that the most serious obstacle to the improvement of these conditions seems to be lack of professional attitude on the part of the teachers. Lacking this, there is little to build upon, either in improving methods or changing aims. This professional attitude may be said to consist of two essential features: First, a broad and thorough knowledge of the subject beyond the borders of the immediate topic in hand; second, and fully as important, a systematic and intimate knowledge of the fundamental principles of modern mathematical pedagogy. Regardless of the reasons why the rank and file of the teachers of mathematics do not possess this equipment, the desire for improvement which would come from such a professional training must be instilled in the teachers before we can expect to find much progress toward better conditions. It is, of course, true that a teacher should be first a man or woman possessed of the highest type of moral character, but if that be the whole equipment, he is not yet a teacher, any more than he is a physician or a lawyer or a preacher. The essential training which will make him work for the improvement of the profession, even when it is hard work, must have as a background that love of his subject and of his calling which can come only through a deep appreciation of what really constitute that subject and that calling. It would seem that a desirable minimum of preparation for a secondary school mathematics teacher should be so much of the subject, at least, as is involved in a first course in the calculus, and so much of pedagogical training as is involved in one year of the history and one year of the principles of education.

With such a groundwork of training for the majority instead of for the very small minority, we should be justified in expecting a professional spirit which would lead to wise and permanent changes tending toward better methods, better aims, and better content in the mathematical curriculum.

In compiling this report, we have supplemented the information furnished by the questionnaires with the best judgment we could obtain from those who have been in close touch with the school systems of the Pacific coast. The present tendencies are not sharply defined, but we are justified in concluding that there is on the part of many teachers a readiness to respond to any sane and thoroughly organized plan for improvement.

SUBCOMMITTEE 7. THE PREPARATION OF TEACHERS OF MATHEMATICS FOR THE PUBLIC HIGH SCHOOLS.

METHOD OF THE INVESTIGATION.

A questionnaire was sent to several hundred city superintendents, high-school principals, and teachers of mathematics. One hundred and sixty-five answers were received, representing 153 cities. Of these, 4 have over five hundred thousand population each, 20 have between one and five hundred thousand, 20 have between fifty and a hundred thousand, 50 have between twenty-five and fifty thousand, 22 have between ten and twenty-five thousand, and 37 have less than ten thousand. These reports represent some 900 teachers of mathematics in all parts of the United States, and are probably typical of conditions in cities whose aggregate population is between fifteen and twenty-five millions. It may be assumed that the large cities have on the whole better educational facilities than the smaller towns. The small towns could not easily be reached.

Letters were also sent to all State superintendents and to many high-school inspectors. Forty-seven replies were received. The character of the information received from them varied so greatly that it did not seem feasible to tabulate it, but it was carefully taken into consideration in making general statements.

The information concerning the facilities offered by our universities and colleges for the training of teachers was obtained by the examination of catalogues for 1909–10.

PRESENT STANDARD PREPARATION OF TEACHERS IN SERVICE.

The amount and quality of school training which high-school teachers in the United States have had varies so greatly with the section of the country and with the size of the school that it is impossible to make any single statement which is strictly true for all. However, there is a very widely prevalent standard which we may take as a point of departure.

It may be said in very general terms that the present standard high-school teacher is a graduate of an unspecialized course in college or university, without pedagogical training or special training in mathematics beyond the ordinary college course. This normally means that the teacher has had eight years in the elementary school, beginning at about 6 years of age, four years in the high school, and four years in the college. In the college the study of mathematics generally stops with a first course in calculus, and often with much less. There are wide variations above and below this standard.

There are six or seven thousand small schools scattered over all parts of the country which employ from one to three teachers, which support a course of from one to three years and which are generally

not on the lists of approved schools issued by the State departments of education or by universities. There are a number of States throughout which the educational standards are relatively low. From these States and from the small schools statistics are not easily obtained. Roughly speaking, they contain about 40 per cent of the high-school students in the United States. It appears that the teachers of mathematics are largely high-school graduates, most of whom have done some work in higher institutions, often normal-school graduates, with a small percentage of college graduates. In a few States the smaller schools generally employ college graduates, but these States are exceptional.

In the remaining schools of the country, containing, roughly, 60 per cent of the total high-school students, the majority of teachers are college graduates, and of those who are not, mostly the older generation, practically all have attended schools above high-school grade for from one to three years. In most parts of the country, even where standards are high, there is a small percentage of teachers of long experience who entered the work before present standards were adopted and who are not college graduates. Reports were received from 152 large schools or city systems, having in all some 900 teachers of mathematics. In these schools, as shown by the reports, 86.5 per cent have the A. B., or equivalent degree. These percentages for the different sections of the country and the number of schools reporting are as follows: New England, 27 reports, 94.5 per cent; Middle Atlantic States, 39 reports, 85 per cent; Central States, 56 reports, 83.5 per cent; Southern States, 12 reports, 79.5 per cent; Western States, 18 reports, 90.5 per cent. In basing conclusions on these figures, it should be noted that in some sections these schools are exceptional, while in others they are fairly typical.

Through the greater part of the United States, systems of high-school inspection and approval are in operation or are being established, either by State departments of education or by universities. The thoroughness of this inspection varies greatly in different States. In approved schools, the percentage of college graduates, calculated for whole States, runs from 75 to nearly 100, varying with the section of the country.

TENDENCIES TOWARD HIGHER STANDARDS FOR TEACHERS.

That standards are being rapidly advanced is indicated by the fact that in by far the larger portion of the country in schools employing four or more teachers and preparing students for the universities new teachers are now quite generally graduates of universities or colleges of good standing, even though many of their older teachers are not. While large numbers of our high-school teachers are graduates of normal schools, that training is no longer regarded as ade-

quate in any large percentage of high schools. The percentage that have taken normal-school courses and have then gone on through their college course is probably quite large.

The fight for the recognition of the principle that high-school teachers should have the training represented by the bachelor's degree is practically won. There is but a small portion of the country where this is not at least a clearly recognized ideal, however remote from realization in practice. It is at this point that the real questions arise which concern the future. What does the college education which they receive mean to the prospective teacher as such? How can its value to him be made greater? What is the next step?

In this connection it is important to know what has thus far been done in the training of high-school teachers of mathematics beyond the unspecialized college course. Of the 900 teachers in large high schools above mentioned the reports indicate that less than 40 per cent have specialized in mathematics in their undergraduate course or have taken a master's degree. The number who have done work in mathematics equivalent to that required for a master's degree by our best universities is small, probably less than 10 per cent even in our best schools, and there are practically none in the medium-sized schools. Those who have the degree Ph. D. or who are doing serious research work in higher mathematics are rarely found teaching in high schools, since there is very little effective demand from the high schools from men of this preparation.

A notable example of a State in which the minimum requirement for high-school teachers is unusually high is California. There the minimum requirement is essentially that the candidate must present evidence that in addition to eight years in high school and college he has done a half year of graduate study in a university belonging to the Association of American Universities and a half year of practice teaching in a high school conducted for this purpose by such a university. These requirements are subject to certain modifications concerning practice teaching which do not materially alter the standard. So far as the committee has learned this is the highest minimum requirement in force throughout any entire State. In several cities minimum requirements practically equivalent to this are in force.

Two special influences have tended to retard the naturally growing demand for higher training, (1) a feeling that the specialist in mathematics does not easily adapt himself to the needs of the high school; (2) a distrust of the pedagogical training of the past. In a word, the separation of specialized scholarship from practical training in the art of teaching has tended to bring discredit on both. This is especially noticeable in the frequent intense and even unreasonable opposition to pedagogical training in the replies received. In this connection the fact should be noted that until very recently only a very few

institutions in this country offered practical training in the teaching of mathematics in connection with sound higher training in the subject matter.

Perhaps the most hopeful sign for the future is the recent establishment in many universities of teachers' colleges and schools of education where special attention is given to the training of teachers for secondary and higher schools in connection with the best opportunities for specialization in the subject matter and for broad general culture. From the high schools, underneath the current criticism of shallow pedagogy and narrow specialization, often expressed in violent opposition to one or the other, there comes an apparent strong desire for sounder scholarship and real efficiency in teaching. Many schools now require that all new teachers shall have had successful experience. As this requirement becomes more general the necessity for the practice school will become still more apparent. In this connection we give a brief statement of the present status of facilities for acquiring special pedagogical training in connection with advanced study of the subject matter.

FACILITIES OFFERED BY UNIVERSITIES FOR THE SPECIAL TRAINING OF TEACHERS OF MATHEMATICS.

On the pedagogical side there has been a remarkable advance in the facilities for the training of teachers of mathematics for high schools in the universities and colleges within the last three years. Three years ago (1907) but 10 out of 30 of our leading universities were offering courses in the teaching of mathematics. To-day 18 of that 30 are providing that instruction, while there are in all 38 universities and colleges giving special attention to such work.

Of these 38, but 11 require practical teaching.

The value of the history of mathematics is becoming recognized, but as yet only 29 colleges and universities are giving explicit courses in this subject. Of course more or less such instruction is given incidentally.

As to opportunities for training in the subject matter nothing need be said here. In all our better universities they are far beyond the present demands of our high schools. Prospective high-school teachers rarely fully avail themselves of the opportunities that are offered. Some universities, about eight in number, offer courses in the logical foundations of mathematics. That more do not offer such courses is probably due to the belief that an appreciation of the logic of mathematics is better obtained in courses in the regular subject matter than in a necessarily elementary course devoted explicitly to the logic of the subject rather than to any lack of appreciation of the importance of training in the logic of the subject. However, waiving this question as beyond our field, the question might well be raised whether prospective high-school teachers, whose training in the

subject matter is necessarily small, might not be benefited by a final course whose object should be to gather together their somewhat fragmentary knowledge into an organized whole, and perhaps to give them some idea of the meaning, purpose, and methods of the higher mathematics which they are not to study.

We restate a summary of catalogue data for 1909–10 by grouping universities into three classes. Column (1) gives the number of institutions that give courses in the teaching of secondary mathematics; column (2), practice teaching in mathematics; column (3), courses in the history of mathematics.

	(1)	(2)	(3)
Members Association of American Universities	13	10	7
Members Association of State Universities	19	14	10
Other colleges and universities	16	2	16
Totals (omitting duplicates)	38	19	29

A very significant advance in this direction is the establishment of a degree with distinction for prospective teachers of mathematics, recently announced by Harvard University in the following terms:[1]

The faculty of arts and sciences have established a degree with distinction in mathematics and education, intended to represent special preparation for the work of teaching mathematics in secondary schools. This degree is to be administered by a standing committee of three under the following rules:

In order to be recommended for this degree the student must have been known to the committee as a candidate during at least the last two years of his course, and he must have carried on his studies in his chosen field under the guidance of the committee.

The degree will be awarded on the basis of the following courses—

1. Mathematics and allied subjects.
   - (a) A course in descriptive geometry or surveying.
   - (b) Three and one-half courses in mathematics above the freshman courses. The choice must include Mathematics 2.
   - (c) Physics *C*, or its equivalent.
   - (d) A course in astronomy.

The student is advised to take both descriptive geometry and surveying. In case he does so the requirement under (*b*) will be reduced to three courses in mathematics. He also advised to include the course in modern geometry (Mathematics 3) among his elective courses. He should in any case consult the chairman of the committee before finally making up his program.

2. Education.
   - (a) A general introductory half-course (Education 2*a*) or a half-course in educational theory (Education 5*a* or 6*a*).
   - (b) A course in the history of education (Education 1).
   - (c) A course in secondary education, with practice teaching (Education 3*b*), or a course in elementary education, with practice teaching (Education 3*c*), or a course in school administration (Education 3*a*).

[1] The University Gazette (of Harvard University, Cambridge, Mass.), May 20, 1910.

The restriction expressed in the words "* * * above the freshman courses" excludes such topics as trigonometry and analytic geometry (introductory course) but admits a first course in the calculus (Mathematics 2) and the elements of mechanics.

Physics *C* is an experimental course in physics for students who have passed in physics for admission to college.

A course is the work implied in following a series of lectures occurring three times a week for about 35 weeks. Four courses, as a usual thing, constitute a year's work for a college student.

The committee reserves the power of exercising an independent judgment in each case; but it must always be satisfied that the program offered furnishes a sufficient basis for distinction, and that the quality of the student's work justifies his recommendation.

DESIRABLE STANDARDS IN TRAINING TEACHERS OF SECONDARY MATHEMATICS.

In this connection we must remember that the majority of those teaching mathematics in our high schools are prepared merely on the side of subject matter, while it is safe to say that in many of our smaller high schools algebra and geometry are taught by those who are not familiar with any mathematics beyond these same subjects.

Where the control of courses of study and the appointment of teachers are left with the local school boards, it is hard to adopt any set standards. By centralized control of secondary schools, it has been possible for Germany and France to maintain a high standard in selecting teachers for their institutions. We can not have national control of education; but by the work of our universities through the inspection of schools, and by State departments of education, much is now being done to elevate standards in the selection of teachers.

What then should we formulate as the proper university training for the prospective teacher of secondary mathematics in the United States? The present tendency gives us the key to an answer. The candidate for a certificate to teach mathematics in our secondary schools should be required to take (1) advanced courses in mathematics, to include at least a first course in calculus; (2) history of mathematics and its bearing on teaching; (3) courses in the general theory of education and in the teaching of secondary mathematics, the latter being given by one familiar with advanced mathematics, the history of mathematics, and the general field of education; (4) observation and practice teaching, in connection with this university training, under the supervision of a specialist in mathematical education. We may hope that candidates be not admitted as full secondary teachers until they have shown after a year's trial that they are specially fitted for the work. Beyond this it is highly desirable that they do some serious work in higher mathematics, including some course which will give them a broad and somewhat unified view of the field. The indications are that real advance in our standards for the higher education of our secondary teachers of mathematics will follow lines of compromise between the work of our best schools of education and the traditional highly specialized course leading to the degree of Ph. D. The pressing need of the hour is that our high schools insist upon, and our universities equip themselves for furnishing, the minimum preparation outlined above. Such a course will in

no wise interfere with the attainment of broad general culture and we believe will furnish the best preparation for the student who is ultimately to teach in the university.

OTHER FACTORS IN THE EFFICIENCY OF TEACHERS.

Summarizing reports from about 165 large high schools or city systems, the following facts are found: Forty-three and a half per cent of the teachers of mathematics are men. The lowest percentage of men teachers, 38 per cent, is found in New England. Two-thirds of these teachers teach only mathematics. In smaller schools not reporting, this proportion is no doubt very much less. Excluding four or five of the largest cities, where the percentage is considerably higher, it appears that about 80 per cent of the mathematics classes are taught by teachers whose chief interest is in mathematics. Teachers teach on an average 26 periods of from 40 to 45 minutes each, the number ranging from 20 periods of 40 minutes each to *40 periods of 45 minutes each.* In New England the average is 23 periods. Eighty per cent of these schools teach trigonometry. No general tendency is discernible toward increasing or decreasing the amount of mathematics taught, individual tendencies about balancing.

The following questions were asked concerning salaries: What is the highest salary per year a mathematics teacher receives as such for full work? What is the lowest? What is the average? The following summary gives the number of cities or high schools which pay the salaries stated at the top of the column:

Number of cities or high schools paying certain salaries to teachers of mathematics.

	Over $2,000.	$1,501 to $2,000.	$1,001 to $1,500.	$751 to $1,000.	$500 to $750.	Total number.
Highest salaries.						
New England	2	5	7	9	2	25
Middle Atlantic	4	12	15	6	2	39
Central	2	3	28	18	1	52
Southern	0	2	6	4	0	12
Western	0	8	9	1	0	18
United States	8	30	65	38	5	146
Lowest salaries.						
New England	0	0	0	8	16	24
Middle Atlantic	0	0	6	16	15	37
Central	0	0	4	31	14	49
Southern	0	0	0	6	5	11
Western	0	1	9	5	0	15
United States	0	1	19	66	50	136
Average salaries.						
New England	1	0	6	10	6	23
Middle Atlantic	0	5	11	10	2	28
Central	0	0	18	20	2	40
Southern	0	0	3	7	0	10
Western	0	2	10	2	0	14
United States	1	7	48	49	10	115

One hundred and thirty-five answers were received from superintendents and principals to the following questions:

In estimating the prospective value of a high-school teacher of mathematics, out of a hundred points, how many points would you give to each of the following?

(1) Knowledge of the mathematics he is to teach.
(2) Knowledge of mathematics of higher grade.
(3) Ability in scientific research.
(4) Pedagogical training.
(5) General education and culture in other lines than mathematics.
(6) Experience in teaching.

The following summary of answers received is self-explanatory:

Question.	Average of answers.	
(1)	36.5	123 answers ranged from 20 to 50.
(2)	11.1	123 answers ranged from 5 to 20.
(3)	5.5	129 answers ranged from 0 to 10.
(4)	11.7	115 answers ranged from 5 to 20.
(5)	16.0	112 answers ranged from 5 to 25.
(6)	19.2	110 answers ranged from 10 to 30.

The question as to whether any inferences of value can be drawn from these is left to the reader.

SUBCOMMITTEE 8. THE SIX-YEAR HIGH SCHOOL.

The traditional school organization of the country consists of an elementary, or grammar, school of eight grades—i. e., years—and a high school of four years. It has been proposed to shorten the grammar-school course to six years, and to incorporate the work of the last two grades, the seventh and eighth, with that of the high school, thus making the high-school course one of six years. The work of the subcommittee is an investigation of the curriculum in mathematics in such six-year high schools.

Excepting the Boston Latin schools, the movement for six-year high schools covers but a few years. The literature is chiefly of a general nature, dealing with the advantages to be gained from such schools, and containing next to nothing in the way of detail that would be of value in a report on the subject under investigation. A bibliography is appended to this report (see p. 94), but for the reason just given no especial pains have been taken to make it complete. The widespread interest taken in the movement is perhaps best indicated by the fact that the National Education Association (N. E. A.) has had a committee considering various aspects of the six-year high schools since 1906 (cf. bibliography, p. 94).

1. ADVANTAGES.

Among the advantages claimed for the six-year high school are:

(a) The use of departmental methods in what are now the last two years of the elementary school. Under the traditional system, pupils in the same grade are taught various subjects by but one teacher, while the six-year plan insures specialists for the teaching of each branch of study two years earlier than at present (but see sec. 2).

(b) The use of laboratories in which the study of elementary science might be begun two years earlier than at present.

(c) The possibility of beginning the study of modern languages earlier than at present.

(d) The transition from the elementary to the high school would be less abrupt than at present. This is insisted on very strongly by advocates of the new scheme.

(e) More pupils would continue their schooling beyond the eight years of the traditional elementary school.

(f) The easier use of manual-training shops and the introduction of industrial-training courses.

(g) In many localities the new plan would possess financial advantages; e. g., it would permit the full use of buildings already erected and relieve the pressure for more buildings for the elementary schools for some years to come.

For a fuller account of advantages claimed see the N. E. A. reports for 1907 and 1909 and the 1909 pamphlet of the New York City Club.

2. APPROXIMATIONS TO SIX-YEAR HIGH SCHOOLS.

(a) Several cities in the country have adopted departmental methods in the seventh and eighth grades, and some of these regard it as a definite step toward the six-year high school, while others think of it as merely a change in the methods of the elementary school. Even in the latter cities several of the advantages claimed for the six-year high school have been enjoyed.

(b) In some cities the high-school course has been lengthened to four and a half or five years.

(c) In some cities there are separate schools for a part of the pupils in the seventh and eighth grades. In these schools the work is correlated with that of the regular four-year high school more closely than in the regular grammar school. Especially bright pupils from these schools may, in some cities, complete the high-school course in three years.

The subcommittee realized at the beginning of the investigation that these various approximations should be included in its study.

3. METHOD OF THE INVESTIGATION.

Following every clue received by correspondence or in the literature resulted in the compilation of a list of 29 cities reported to have a fair approximation to the six-year high school and of 27 cities in whose schools departmental methods are said to be employed in the seventh and eighth grades. While the latter list contains all cities of the sort indicated which came to the attention of the subcommittee, no especial effort was made to compile a complete list. This was due to the fact that three important cities reported by the Bureau of Education at Washington to contain schools belonging to the second class were investigated personally by different members of the subcommittee during the last summer (1909) and found to offer nothing of value for this report.

The subcommittee is indebted to the courtesy of Mr. Charles S. Hartwell, 234 Willoughby Avenue, Brooklyn, N. Y., secretary of a joint committee of the teachers' associations of New York City and Brooklyn conducting an extensive investigation of school organization, for nearly all the cities on both lists, and for many helpful suggestions.

In advance of the information that the Bureau of Education had offered to circulate questionnaires, the subcommittee mailed a circular letter to the superintendent of schools in each of the cities on the two lists, with a request that an inclosed letter be handed to a teacher of mathematics. Later a copy of the second letter, somewhat modified, was mailed to the department of mathematics of the high school in each of the cities on the first list from which no reply had been received. The replies from the cities on the second list indicated that it was not worth while following up those not answering the first letter. Conditions in a few cities were investigated personally, and no letters were mailed to these cities.

4. ANALYSIS OF THE INFORMATION RECEIVED.

The number of schools investigated is so small, and their forms of organization and curricula so varied, that it is impracticable to tabulate the information received. Instead, the report can only present a summary of the situation in each school (sections 6–11), and a few general comparisons (section 13). The value of the latter must be estimated in the light of the fact that the number of schools is small and that the conditions in different parts of the country vary a great deal.

The subcommittee is able to report upon:

(a) Five schools organized on precisely the lines indicated in section 1. (Section 6.)

(b) Two schools approximating that organization very closely. (Section 7.)

(c) Two five-year high schools. (Section 7.)

(d) One four-and-a-half-year school. (Section 7.)

(e) Four cities having special schools preparatory to high school. (Section 8.)

(f) Six cities having departmental methods in the seventh and eighth grades (section 9), but not all of these regard this as an approximation to a six-year high school.

(g) Two have a different form of six-year high school. (Section 10.)

(h) One noteworthy private high school. (Section 11.)

Neglecting (f), (g), and (h) the subcommittee can report upon but 14 six-year public high schools or approximations thereto.

The cities on both lists not covered in the above classification were either not heard from at all or offered little of value to the subcommittee.

Cities employing departmental methods in the seventh and eighth grades have not, so far as information at hand shows, made much effort to mold the mathematics in these grades and in the high school into a homogeneous whole, and hence, contrary to hope, they furnish little of value to the subcommittee.

5. THE TRADITIONAL CURRICULUM.

For a proper appreciation of the following sections it should be understood that the traditional curriculum has been, roughly, arithmetic in the seventh and eighth grades of the elementary school, elementary algebra (i. e., algebra through quadratic equations, arithmetical and geometrical progessions, etc.), and plane geometry in the high school. Most high schools also offer solid (including spherical) geometry, advanced algebra (i. e., such topics as permutations, logarithms, theory and numerical solution of equations, etc.), and plane trigonometry.

Solid geometry, advanced algebra, and trigonometry are required for admission to technical schools but are usually taught in the first year of college work. Hence students preparing for college do not usually take these subjects in the high school. In the West, solid geometry is usually required for admission to college.

These subjects have usually been taught with very little attention to their interrelations, and many students take mathematics during but a portion of the four years in high school; e. g., a student may take elementary algebra the first year, plane geometry the second, no mathematics the third year, and a review of one or the other of these subjects in a portion of the fourth year.

Abbreviations used in sections 6–12:

Arith. for arithmetic.
Alg. for elementary algebra.
Geom. for plane geometry.
Trig. for plane trigonometry.
Adv. Alg. for advanced algebra.
Obs. Geom. for observational geometry.

The six years of the course are named uniformly in this report, but usage varies in different schools.

6. SIX-YEAR HIGH SCHOOLS.

Boys' Latin School and *Girls' Latin School*, constituting The Public Latin School in Boston, Mass., founded 1635. This has been a six-year school for a great many years, preparing students for college but not for technical schools.

First year: Arith. 3,[1] Obs. Geom. 2.

Second year: Arith. 3 ($\frac{1}{3}$ year), Alg. 3 ($\frac{2}{3}$ year), Obs. Geom. 1 (the entire year).

Third year: Alg. and Obs. Geom. (correlated) 4.

Fourth year: Alg. 3 ($\frac{1}{2}$ year), Geom. 3 ($\frac{1}{2}$ year).

Fifth year: Geom. 3 ($\frac{1}{3}$ year), Alg. 3 ($\frac{2}{3}$ year).

Sixth year: Plane and Solid Geom. 4.

Recent changes, as indicated by printed reports, are: The placing of algebra as early as the second year and of formal geomtry as early as the fourth year; the teaching of some geometry in each year. But, essentially, this curriculum has been in operation for some years.

Roxbury Latin School, Roxbury (Boston), Mass., founded 1645.

First year: Arith. (a little Alg.) 3.

Second year: Arith. 2 ($\frac{1}{3}$ year), Obs. Geom. 2 ($\frac{2}{3}$ year).

Third year: Alg. 4.

Fourth year: Alg. 3 ($\frac{2}{3}$ year), Arith. 3 ($\frac{1}{3}$ year).

Fifth year: Alg. (a little Adv. Alg.) 3.

Sixth year: Geom. 5.

Adv. Alg., Solid Geom., Trig. (electives).

Lead, S. Dak.—The course in mathematics covers but five years. Exceptionally bright pupils may do all the work of the six-year course in five years. The six-year course has been in effect five years.

First or second year: Arith. 5.

Third year: Alg. 5.

Fourth year: Geom. 5.

Fifth year: Alg. 5 ($\frac{1}{2}$ year), Solid Geom. 5 ($\frac{1}{2}$ year).

Sixth year: Adv. Alg. 5 ($\frac{1}{2}$ year), Trig. 5 ($\frac{1}{2}$ year).

[1] The number is that of recitations per week, when it is known. The recitation period is usually 40 or 45 minutes.

Algebra was formerly taught in the second year, but has been put back into the third year.

Hope Street English and Classical High School, Providence, R. I.—The report of their curriculum is incomplete.

First year: Arith. (probably).

Second year: Alg. 3.

Last four years probably Alg., Geom., Solid Geom., Adv. Alg., and Trig.

Principal Charles E. Dennis, jr., writes:

"Correlation consists in not emphasizing the advanced portions of arithmetic, i. e., geometrical measurements, and omitting cube root entirely. These subjects are better taught in connection with algebra and geometry."

Remarks.—Four of the five schools introduce some algebra in the first two years. In the fifth, algebra has recently been dropped from the second year.

The three Latin schools introduce some geometry in the first two years. They also provide for a continuous course of six years.

Two of the five schools do not offer trigonometry.

In at least four of these schools the traditional "tandem method" of arranging courses has been somewhat broken up. This breaking up is especially noteworthy in the Boston Latin schools.

7. CLOSE APPROXIMATIONS TO THE SIX-YEAR HIGH SCHOOL.

(Five-year and four-and-a-half-year high schools.)

Crawfordsville, Ind.—"The seventh and eighth grades and the high school occupy the same building in our city and all are organized on the departmental plan. There is a slight division between the eighth-year course and high school, but it is very slight. We consider the arrangement an approximation to a six-year high-school course."

First year: Arith.

Second year: Arith. and Alg.

Last four years: Alg. (1½ years), Geom. (1 year), solid Geom. (½ year).

Richmond, Ind.—Information on Richmond is due to the kindness of Mr. Hartwell, who kindly sent the subcommittee a copy of a letter from the superintendent of schools.

For twelve years the seventh and eighth grades have been housed in a separate building in which departmental methods are used. The heads of the departments are the heads of departments in the high school.

"We are much pleased with this plan. We are hoping to be 'brave' enough to print our high-school course as including the six grades *in name* as well *as in fact*."

First year: Arith.

Second year: Alg. (½ year).

Last four years: Probably Alg., Geom., Solid Geom., Adv. Alg., and Trig.

The Muskegon High and Hackley Manual Training School, Muskegon, Mich.—A five-year high school. "The seventh-grade work in our city schools is given in the ward schools in a departmental system. The eighth-grade work is given at the high school, making with the regular four years' work a course of five years."

First year (seventh grade, not in five-year course): Arith.

Second year: Arith. (½ year), Alg. (½ year).

Third year: Alg.

Fourth year: Geom.

Fifth year: Alg. (½ year), Geom. (Solid?) (½ year).

Sixth year: Trig. and Alg.

"In our first two years of algebra we cover about the same ground usually covered in 1½ years in other high schools."

"We are contemplating extending the one-half year of arithmetic to a whole year in the eighth grade, perhaps retaining a little introductory work in algebra."

"Due to our extended work in manual training we require mathematics only through plane geometry for most students."

Woodward Avenue High School, Kalamazoo, Mich.—Approximately a five-year high school.

"Our seventh grade is not departmental. The eighth-grade program is arranged on the same plan as the high school: Pupils sit in a grade room where they keep their books and to which they go after each class. As the class bell calls, pupils go to their respective classes, or if they have no class, to the general study room for eighth grade and high school."

First year (seventh grade, not in five-year course): Arith.

Second year: Arith.

Third year: Alg., Arith. (½ year, elective).

Fourth year: Alg. (½ year), Geom. (½ year).

Fifth year: Geom. (probably plane and solid, elective).

Sixth year: Trig. (½ year), Alg. (½ year) (both elective).

Pittsburg, Kans.—A four-and-a-half-year high school, organized nine years ago.

First year (seventh grade, not in four-and-one-half-year course): Arith.

Second year (last half in four-and-one-half-year course): Arith.

Last four years: Probably Alg., Geom., Solid Geom., Adv. Alg., and Trig.

Remarks.—In three of these five schools algebra is introduced in the first two years and no geometry is given in these years in any of the five.

8. APPROXIMATION CONSISTING OF A PREPARATORY HIGH SCHOOL AND A REGULAR HIGH SCHOOL.

Worcester, Mass.—Special schools for a portion of the pupils in the seventh and eighth grades and a four-year high school.

First year (seventh grade): Arith.

Second year (eighth grade): Arith. (½ year), Alg. (½ year).

Third year: Alg.

Fourth year: Geom.

Fifth year: Adv. Alg. and Solid Geom.

Sixth year: Review of Alg. and Geom.

Superintendent of Schools H. P. Lewis writes: "I believe thoroughly in a six-year high-school course in which the curriculum would be different in many points from that of the last two years of the elementary course and the present four years of the high-school course."

Grand Rapids, Michigan.—Superintendent of Schools William A. Greeson writes: "In two buildings we have the departmental organization. * * * The curricula in these grades (the seventh and eighth) have not been closely correlated with those in the high school."

It is inferred from other portions of his letter that these buildings are used exclusively for the seventh and eighth grades.

"As soon as our two high-school buildings are completed and occupied by the high schools, we shall have two buildings * * * which can be used for the seventh, eighth, and ninth grades. It is our purpose then to organize these three grades upon a modified high-school plan."

First year: Arith.

Second year: Arith.

Third year: Alg.

Fourth year: Alg. (½ year).

Fifth year: Geom.

Sixth year: Solid Geom. (½ year), Adv. Alg. (½ year), Trig. (½ year).

Lincoln, Nebr.—There is a five-year course consisting of: First, a course of two years, known as preparatory to the high school, for especially bright pupils, which covers, with some omissions, the work of the seventh and eighth grades and that of the first year of the regular four-year high school. Second, the last three years of the high school.

First year: Arith. and Obs. Geom.

Second year: Arith.

Third year: Combined with first two.

Last three years: "A completely blended course of algebra and geometry."

There is no mathematics in the first year of the regular four-year course except in the commercial department. What is taught there is presumably arithmetic.

Baltimore, Md.—Children with a sufficiently good record in the sixth grade of the elementary school may attend any one of four schools doing work in the seventh and eighth grade preparatory to high school. A good student may complete these grades and the work of the regular four-year high school in five years. The curriculum is probably the traditional curriculum. (Section 5.)

Remarks.—One of these four schools introduces algebra in the first two years and but one introduces geometry.

9. APPROXIMATIONS CONSISTING MERELY OF DEPARTMENTAL METHODS IN THE SEVENTH AND EIGHTH GRADES.

Aurora, Ill.—Superintendent of Schools C. M. Bardwell writes:

> Our seventh and eighth grades are distinct from the high school, as in case of most other systems. The accommodations necessitate this.
>
> For a number of years we dropped the subject of arithmetic at the end of the sixth grade, and took for the next half year inductive, or constructive, geometry! We began algebra the second semester of the seventh grade and carried it through the eighth grade, with recitations five times a week. We have changed this, however, in the last year and have introduced arithmetic twice a week, carrying over the subjects that we had formerly had in the sixth grade, but omitting most of the technical applications of arithmetic.
>
> The work which we give the children in mathematics in these grades (the seventh and eighth) is not so much with a view to fitting them to some special course in the high school, or a hasty completion of regular mathematical work, but rather because it seems to fit their needs from a standpoint of their intellectual growth. They seem to enjoy their work in algebra intensely, and act as though it were something that they really feel the need of and a desire for.

Saginaw, Mich.—This city is popularly reported as being a pioneer in the six-year movement.

First year: Arith. 4.

Second year: Arith. 4.

Third year: Alg. 5, Commercial Arith. 5, Industrial Arith. (in process of evolution).

Fourth year: Alg. 5 (½ year), Geom. 5 (½ year).

Fifth year: Geom. 2.

Sixth year: Alg. (some Adv. Alg.) 3 (½ year), Solid Geom. 3 (½ year), Trig. 3 (½ year), Surveying 3 (½ year).

In the fourth year, geometry is studied without a text.

Herkimer, N. Y., and *Franklin, Ind.*—These schools have the traditional curriculum (section 5) except that the latter has no trigonometry.

Iron Mountain, Mich.—"We formerly had departmental work in the seventh and eighth grades for eight or nine years, but have partly abandoned it and entirely so in mathematics."

"A little algebra and geometry" is introduced in the eighth grade.

Algebra and geometry are "mixed" in the last two years.

Indianapolis, Ind.—Departmental methods are not regarded as an approximation to a six-year school. The city is mentioned chiefly on account of the fact that correlation has been highly developed in the high school and on account of the following excerpt from a letter from Mr. W. W. Hart, head of the department of mathematics, Shortridge High School:

"I might say that it is our intent to have very little algebra in the eighth grade in the future on the ground that such work does not seem to be of sufficient practical value to the boy and girl who does not go to the high school."

10. A SECOND FORM OF SIX-YEAR HIGH SCHOOL.

A movement has developed in the Middle West for the introduction of two years of graduate work in the high school, covering approximately the first two years of college work. In mathematics this necessitates the teaching of analytic geometry and differential and integral calculus in addition to all the courses of the traditional curriculum. (Section 5.)

Joliet, Ill., and *Goshen, Ind.*, have organized high schools on this basis. In the latter city the seventh and eighth grades are organized departmentally. Bright pupils may begin algebra in the eighth grade, but otherwise the curriculum does not differ from the traditional curriculum.

11. THE HORACE MANN SCHOOL.

This high school is connected with Teachers College, Columbia University, and is intended to be a model high school. It is not a public high school in the sense that there are no tuition charges. Some departments offer six-year courses as well as a four-year course. The department of mathematics offers, in addition to a four-year course, the following five-year course:

First year: (Not in the course.)
Second year: Alg.
Third year: Alg.
Fourth year: Geom.
Fifth year: Alg. (½ year), Solid Geom. (½ year).
Sixth year: Trig. 3 (½ year), Adv. Alg. (a little analytic Geom.) (½ year).

12. A PROPOSED CURRICULUM.

In a paper in the Educational Review, vol. 25 (1903), pages 455–63, Prof. Paul H. Hanus proposes a readjustment of the entire education system of the country. He proposes (p. 458) a six-year course for two high schools of three years each, as follows:

First year: Alg. 2, Obs. Geom. 2, Arith. 1.
Second year: Alg. 2, Obs. Geom. 2, Arith. 1.
Third year: Arith. 3, Obs. Geom. 2.
Fourth year: Alg. 2, Geom. 3.
Fifth year: Alg. 4, Mechanical Drawing 1.
Sixth year: Solid Geom. 5, Trig. 5.

13. GENERAL CONCLUSIONS.

In conclusion, the subcommittee can do little in the way of general remarks. Comparing the 14 schools considered in sections 6–8, the following variations from the traditional curriculum may be noted:

One school teaches algebra and three teach geometry in the first year.

Seven teach algebra and three teach geometry in the second year.

Four schools have correlated the arithmetic, algebra, and geometry sufficiently to break up effectively the "tandem method" of arranging courses. Information at hand is not sufficiently detailed to indicate how much correlation, if any, is attempted in many of the other schools, but the presumption is that there is but little.

Three of the six schools in section 9 present variations from the traditional curriculum in some of these ways.

It is thus seen that the tendencies to insert some algebra and geometry in the last two grades of the elementary school, and to weld arithmetic, algebra, and geometry into a unified course in mathematics, tendencies which have been much discussed in recent years, are being tried out more or less thoroughly in the schools whose organizations approximate the six-year high school. So few schools, however, replied with a description of their courses by topics, as requested by the subcommittee, that it would be useless to attempt to tabulate what little detailed information has been received. The Boston Latin School stands foremost in exemplifying both of these pronounced tendencies.

Further information on these tendencies in schools organized on traditional lines will doubtless be found in the reports of other committees and subcommittees engaged in this investigation.

There has also been much discussion in recent years concerning the introduction into the high school of essentially new matter, such as the elements of analytic geometry and the calculus, but none of the schools investigated have done this (except the two in section 10 which teach the first two years of collegiate work).

While one vigorous protest against the movement was received (Bloomfield, N. J.), the general tone of the letters received by the subcommittee indicates an enthusiastic belief in the six-year high school.

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SUBCOMMITTEE 9. FAILURES IN THE TECHNIQUE OF THE TEACHING OF SECONDARY MATHEMATICS: THEIR CAUSES AND REMEDIES.

INTRODUCTION.

In this commercial age, which makes efficiency the controlling factor in all walks of life, the teacher must not expect to escape a searching scrutiny of his results or criticism of his methods. The general public has at last begun to watch with interest, not entirely free from meddlesome curiosity, the work of its tax-supported schools. It is well that it should be so. For may not many of the educational crises of the past be traced to the aloofness of the average teacher from the active world? Separated from the noise and the merciless competition of real life, the school frequently fails to make those unavoidable readjustments which a business establishment effects almost automatically in its effort to remain "up to date."

Thus it happens that the educational process so often is "behind the times." With majestic inertia the school system glides on in its

accustomed and "approved" course, long after the familiar landmarks have disappeared and the compass needle points to strange and untried seas. Finally there are dangerous collisions and the passengers complain of the wearisome, aimless trip. Then the educational pilots are roused from their stupor and frantic efforts are made to ascertain the "new course." A "reform wave" is suddenly espied, and carried by this "new movement" the distressed craft tries to regain its bearings.

Another reason for this lack of adjustment lies in the difficulty of the educational process. A new machine is easily installed. A mechanical improvement can readily be tested. Not so the infinitely subtle machinery of the mind. A well-known psychologist frankly admitted that in regard to many aspects of the educational problem psychology is as silent as a sphinx. Scientific pedagogy is only in its infancy. It will be found that in most cases the successful teacher, cautioned of course by scientific study against obvious blunders, rises on the basis of sympathy and tact to the experimental acquisition of a satisfactory technique. But all experimenting takes time. Unless undertaken with the utmost care, it is almost sure to mean educational waste. To this should be added the fact that the average teacher holds office for a short period only and that many schools have practically new faculties every year. The new and inexperienced teacher either follows the rut left by his predecessors or indulges in experimentation of a more or less doubtful character. Only a small percentage of secondary teachers have specialized in their work sufficiently to become really creative.

Thus we have a constant oscillation from stagnation to frantic reform. And it is difficult to determine which is more amusing: The abyssal, cocksure self-complacency of the orthodox old-timer, or the innocent glee of the reformer who announces a new patent remedy for all educational ills. What we really need is a less jerky and erratic development, less dangerous stagnation and more genuine progress.

PURPOSE OF THIS REPORT.

The present inquiry was undertaken with a view to answering the following questions:

(1) Are the results obtained in the teaching of secondary mathematics satisfactory?

(2) If not, is mathematics taught more poorly than other high-school subjects?

(3) In case it is taught as well, and yet the results are poor, what general or specific causes of failure can be pointed out?

(4) What remedies can be suggested?

METHOD OF INVESTIGATION.

The subcommittee first sent out about 120 circular letters. These brought about 40 replies. Then a somewhat modified form of the first circular was sent out to a large number of high schools by the committee on public high schools, of which this subcommittee is a part. This second effort resulted in more than 80 additional replies, bringing the total up to about 125. A detailed summary of these answers will be found at the end of this report. Many important educational centers responded splendidly, especially New York and Philadelphia. It appears from these answers that the teachers of mathematics do not believe that their subject is taught more poorly than other secondary subjects.

A second source of information was furnished to the committee by the statistics of two examining bodies, the Regents of the State of New York and the College Entrance Examination Board. It may be argued with perfect candor, of course, that examinations are a very unreliable criterion of pedagogic conditions, and that examination results are often inversely proportional to good teaching. However, these results will furnish some comparative data of great value. Hence we feel justified in submitting the subsequent statistical tables.

In the third place, we must mention the many valuable contributions to a solution of our problem in the educational literature of the past 10 years, in the reports of associations of teachers, etc. So vast is this material that the committee assimilated only a small portion of it. Excellent reports and discussions may be found in recent volumes of School Science and Mathematics, the School Review, the Mathematics Teacher, etc. It can not be said that the comments found in these journals are altogether reassuring. There is constant complaint of educational waste and inefficiency.

STATISTICS OF FAILURES.

The records of the Regents' Examinations and those of the College Entrance Board (for statistics, see Appendix), seem indubitably to warrant the following conclusions:

(1) Mathematics is not taught more poorly than other secondary branches.

(2) The results are not satisfactory. The second conclusion would be even more obvious if the percentage of failures included the pupils who dropped out during their course, i. e., if it were based on the initial registration of the school. Evidently from 40 to 50 per cent of all students pursuing mathematical work in secondary schools do not complete their work satisfactorily, if examinations can be trusted to determine the pupil's mental equipment.

Views of individuals.—Having considered the objective verdict of statistical records, we turn now to the more or less subjective views of

individual teachers and schools. Dr. N. J. Lennes writes (The Mathematics Teacher, March, 1909):

> One of the most obvious facts about mathematics in our secondary schools is a very general dissatisfaction which is expressed on all sides. There is an alarming number of failures, especially in the first year of the high school, which argues that the pupils do not find the subject suited to their tastes and capacities. Instructors in the colleges and universities rarely miss an opportunity for declaring that their students come poorly prepared. The programs of teachers' meetings and the tables of contents of pedagogical journals are teeming with titles which assume that something is wrong.

The college side of the question is discussed at some length in the Cornell Register (1909–10, p. 43). After stating that the preparatory work in mathematics should equip the student with (1) a "certain degree of mathematical maturity," (2) "an accurate and ready knowledge" of specific facts, it continues as follows:

> On the other hand, most students who fail in their university mathematics fail because they are poorly equipped in the second requirement above mentioned. For example, they can not perform the ordinary operations of algebra either rapidly or accurately, they do not know the theory of quadratic equations, they are lost among trigonometric formulas, and they blunder when they use logarithms. Instead of spending their time and energy upon their new work, they must spend much of it in studying up those things with which they ought to be familiar, and thus handicapped they can not keep up the pace set by men who are properly prepared, and they can not do the work that must be done to fit them for the professional work that follows.
>
> It is not sufficient that the student should once have known his preparatory mathematical subjects; he must know them at the time when he begins his work here. It seems absolutely essential, therefore, that these subjects be very carefully reviewed just prior to entrance.

The general dissatisfaction referred to above has induced some alarmists to put mathematics on the list of elective subjects. How characteristic a blunder! As if mathematics, the foundation par excellence of our scientific era, could by a mere stroke of the pen become an optional study, merely because many teachers do not know how to make it palatable, or because our ill-arranged curricula can not accomplish wonders, or because so many of our boys and girls have been allowed to follow the line of least resistance. For a discussion of this elective tendency we must refer to the Third Report of the Association of Mathematical Teachers in New England (Boston, 1906).

CAUSES OF FAILURE—PRELIMINARY ANALYSIS.

If we turn now to a consideration of the causes of failure, it becomes at once apparent that our problem is both general and specific. General, in that mathematics is not alone in its inferior results, a fact proved beyond dispute by statistics. Specific, in that we must determine the particular aspects which this general deficiency assumes in the mathematical classroom, as well as the particular remedies that mathematics may offer for overcoming the general weakness. The

individual teacher, when confronted by a large number of failures, rarely takes a sufficiently broad view of the situation. In some cases he frankly blames his own limited preparation, his prosaic and uninteresting methods, his lack of enthusiasm. Usually it is the pupils who are condemned in toto, or it is the school system or any number of other factors.

There is little doubt that nearly every high school subject is taught with clearer perception of its aim, methods, and content than a generation ago. It is even claimed by some that results are better than formerly. (Cf. Report on the Norwich tests, School Review, May, 1910.) If in spite of this improvement and this increased effort our expectations are so poorly realized, must we not first of all look for underlying causes and conditions beyond the control of the average teacher or even the average school?

It has been said over and over again that we are living in a transition period. Old standards are being replaced by uncertainty and scepticism. Education is powerfully affected by this general unrest. Hence, any consideration of the causes of failure would be incomplete and meaningless without a study—however brief and imperfect—of the influences that are transforming our educational sytem.

TRANSFORMING INFLUENCES IN EDUCATION.

For a brief summary of the present tendencies in education we may refer to a paper read by the chairman of this committee at the Cleveland meeting of the N. E. A. (published in School Science and Mathematics, November, 1908). The following quotations are taken from it:

1. In the first place, modern industrialism, with its demand for tangible success, has led to a great outcry for more practical school work. There is an increasing contempt of "mere theory." This feeling finds its expression in the establishment of trade and technical schools. Mathematics, as usually taught, furnishes a welcome target to the utilitarian educator. As a result there is a growing fear that we may drift too far from the ideal of liberal culture and that the direct bread-winning power of a subject may be made the sole criterion of its usefulness.

2. Our large cities, the natural centers of industry, are also becoming great centers of population. Naturally the struggle for existence is becoming keener. Many parents are now sending their children to the high school to fit them, in the briefest possible time, for a more comfortable life than they themselves enjoy. This has made the high school population more diversified than ever before, and the demands imposed upon the schools have become more numerous from year to year. *For the first time in history, secondary education is truly democratic.* But it can not be denied that the assimilation of so much raw material from homes giving no cultural impulses, and of so many students having no intention of entering higher institutions of learning, is one of the most serious problems of the high school.

3. More far-reaching than these changes of ideal and environment have been certain revolutions in school curricula and methods of instruction. The natural sciences have risen from comparative obscurity to great prominence. Their inductive method of investigation is considered by many as the great panacea for all our troubles. The

influence of the laboratory method is undeniable. It is reacting, for example, on the teaching of history and the languages. In so far as it insists on self-reliance and definiteness of results and is productive of greater interest, it is excellent. But it lengthens school hours, calls for costly equipment, and demands much outside work on the part of pupil and teacher.

4. It would be difficult, moreover, to overestimate the effect of the "new education." Its fundamental precept that all work must be arranged psychologically and adapted strictly to the child's power of comprehension is eminently sound. But it has also given us the enriched curriculum, and the doctrine of interest which replaces all objective standards by the subjective attitude of the child. Unquestionably this means at once a distinct advance and a very real source of danger. The complaint is not infrequent that in many cases the young are learning to depend too much upon the inspirational powers of the teacher, that all real difficulties are carefully avoided, and that the very aim of all true education, to develop a strong character and to create self-activity and initiative, is thereby defeated.

During the past five years we have heard much of the social function of the school. Playgrounds, evening schools, social centers, school clubs, a multitutde of new school activities, claim the attention of teachers and pupils. A prominent professor kindly informed us that until exercises in spelling, mental arithmetic, and formal grammar should have become merely incidental and subsidiary, the high-water mark in teaching would not have been reached. It is exaggerations of this sort that rob many otherwise excellent ideas of their legitimate influence and place upon them the stigma of the faddist.

A corollary of this new gospel of social efficiency is the new doctrine of mental discipline. Although not yet clearly formulated, its main contentions are: (1) That mental discipline as ordinarily conceived is a myth, in the sense that no "general training" is to be derived from the intensive study of one or more subjects, such as Latin, algebra, etc.; (2) that the disciplinary value of a subject is a function of the interest which it inspires and of the motive guiding the student; (3) that the cultural value of a subject depends on the extent to which that subject can be, and actually is, linked with the activities and the thought content of real life. In this way the old static, historic, idealistic conception of mental discipline is being replaced by a dynamic, realistic, practical view. (Cf. Formal Discipline, by C. J. C. Bennett, in Teachers College Series, Columbia University.)

It is not so much this new theory itself, as the hasty inferences drawn from it by superficial minds that we must regard as dangerous. In the first place, it requires no proof that the mere completion, however mechanical and stereotyped, of so much "prescribed" Latin or mathematics does not make an educated person. It is equally true that not all boys and girls find the old school subjects profitable, and for a certain number of them industrial or commercial studies are preferable. This proves nothing concerning the presence or absence of inherent disciplinary value in the present curriculum. The early and onesided introduction of professionalism, no matter how successfully managed, is always deplorable. Wherever it becomes imperative, it should be looked upon as a necessary evil.

For "it is difference in culture, far more than difference in wealth or position, which separates man from man, and class from class." The blending of liberal education and technical training is the supreme educational problem of our day. In Europe it has been solved by a differentiation of schools. America may have to follow that plan. Secondly, the new theory of discipline does not imply the cultural equivalence of all subjects. On the contrary, it demolishes that view completely. If it can be shown that mathematics can be linked with a larger range of actual thought processes and activities than typewriting or bookkeeping, for example, then the greater disciplinary value of mathematics will have been established. Hence, instead of crowding out the "old studies," the new conception of mental discipline simply gives a better criterion for testing their value and should have the effect of securing better training.

Last, not least, the mathematical curriculum has been affected powerfully by two other, mutually opposing, forces. The Perry movement, a direct outgrowth of the trend toward more practical mathematics, aims to abolish from the course all unnecessary details and to substitute experimental verification for logical deduction. On the other hand, the tremendous progress of scientific research, effected by men like Pasch, Peano, Klein, Hilbert, Veronese, Poincaré, has called attention to the many flaws in the logic of our textbooks and has given new impetus to the demand for genuine mathematical rigor.

GENERAL CAUSES OF FAILURE.

All these transforming influences if overlooked or ignored by the teacher may become prolific sources of failure. The constant shifting of the educational background demands on the part of the teacher sound judgment, quick insight, wide training, and the capacity for sane, conservative, readjustment. The new situation makes each educational factor both an active and a passive participant in the educational process. The latter distinction is important, as it should save many teachers from unnecessary discouragement. We shall proceed to examine briefly the principal educational factors as causes of failure, either in an active or in a passive rôle.

1. The Teacher.

Granted that the teacher meets all the obvious preliminary requirements, such as a strong and yet sympathetic personality, tact, enthusiasm, he nevertheless frequently becomes a serious cause of failure through any one of the following factors:

(1) *Lack of professional preparation.*—This is perhaps the most vital and distressing point of weakness in American secondary education. The only hopeful thing about it is that we are beginning

to feel this weakness. Much pounding was required to rouse us. At first it was the foreign critic, or the acclimated American, who put his finger on the sore. Who has not read Prof. Münsterberg's scathing criticisms? And now every number of the educational magazines sings the same melody. "Wanted—a teacher," exclaimed James H. Canfield, of Columbia University, 10 years ago (Educational Review, December, 1900). Some reported cases of professional ignorance seem almost incredible. Time was when everybody thought he could teach everything. This miserable Jacotot fallacy gradually dominated American education, because it harmonized so splendidly with the American spirit of self-activity and independence. Mr. E. George Payne, in a report published by the Kentucky Department of Education, 1909, tells of a young woman who planned to make her entire preparation to teach German in "one of the leading high schools" in Kentucky by spending six weeks on the subject at a summer school. Then he says, "I insist that 90 per cent of those attempting to teach the modern languages in the American schools, especially in the Kentucky schools, do not perform better work than this lady did in first-year German." (See School Review, June, 1910, p. 433.) Shocking, if true. New York State three years ago had only 32.2 per cent of college graduates among its 4,668 secondary teachers. This means that the remaining number had only the equivalent of a high-school education in mathematics. Their horizon was only a little above that of their pupils. How many high-school teachers of mathematics at the present day have ever studied analytics, not to speak of the calculus? How many have ever seen Crystal's Algebra, or Hilbert's Foundations of Geometry, or have read the pedagogical works of Smith or Young? All honor to the noble men and women who, in spite of serious handicaps, have done good work. But every effort should be made from now on by individuals and by schools to secure better professional equipment.

(2) *Lack of professional contact.*—Owing to the vast extent of the country it has been hard to develop esprit de corps outside of the big centers. Departmental organization in the large schools was the first step in the right direction. Associations of teachers soon followed, and any ambitious teacher can now make it possible to meet his colleagues at the educational gatherings. For those who can not go to these meetings the printed reports published in the new mathematical journals furnish a substitute. Lack of contact has been and still is a great source of stagnation. Let us hope it may speedily be removed.

(3) *Overwork.*—It need hardly be said that any teacher who has more than 25 periods a week of required work can not do that work effectively. Entirely indefensible is the practice of loading on a teacher three or four different subjects, especially if they are entirely

"out of her line." Careless, half-hearted, indifferent teaching or incessant worry is the inevitable result in most cases. Many classes are altogether too large. We hear so much about individual instruction and personal attention and yet expect a teacher to inspire 40 young people at a time. So long as these conditions prevail to any considerable extent no improvement can follow.

(4) *Short and unstable tenure of office.*—Another very real source of failure: There is altogether too much shifting of positions. It takes several years to grow into a new position. A teacher who finds herself at a different school every two or three years can not expect to be of great service to that school. A very large number of teachers drop out every year, either to be married or to begin a different occupation. At the 1909 meeting of the American Association for the Advancement of Science Prof. William C. Ruediger, of George Washington University, presented the results of some investigations of the qualities of merit in teachers. He claimed that the best teachers had taught an average of *14 years* and the poorest *8 years*. No teacher who ranked first or second had taught less than 5 years. (See Science, for Apr. 15, 1910.) The shortness of the average teacher's service is due largely to the appallingly low salaries and to the uncertainty of tenure of office. Salaries often are not up to the standard of even the street laborer. No great improvement in teaching need be expected until we shall have (1) better salaries, (2) permanent appointment after a limited trial period, (3) a pension guarantee after a fixed term of service.

2. The Pupil.

So long as human nature is imperfect the problem of the pupil will remain with us, especially during the critical period of adolescence. However, many of the difficulties besetting the teacher would all but disappear if it were not for the following great causes of failure:

(1) *Immaturity.*—This is an entirely indefensible factor, in view of the work done in European schools by pupils of the same age. The enrollment of the Chicago public schools in January, 1909, showed the following arrangement of ages:

High school.	Ages.										Total.
	11	12	13	14	15	16	17	18	19	20+	
First-year pupils	2	45	708	2,291	2,104	1,045	257	60	14	19	6,555
Second-year pupils	0	2	47	467	1,236	1,062	532	159	27	14	3,546

(2) *Lack of preparation.*—A large majority of the teachers reporting to us emphasized this point. It seems that our elementary schools either find the problem of democratic education too big a

task or use methods that do not produce lasting results. A very large percentage of children do not complete the school course (Chicago, 1909: First grade, 38,239; eighth grade, 14,795).

(3) *Aimlessness.*—The "life-career motive," discussed by President Eliot at the 1910 meeting of the National Education Association, is becoming more essential in proportion as increased competition demands greater technical or professional equipment. Altogether too many pupils have no dominant purpose that might keep them at work. A moderate vocational tendency may prove a partial remedy, although it would be a grievous mistake to make vocational studies the sole basis of our secondary education.

(4) *Social diversions.*—Pupils must, of course, have a certain amount of relaxation. But many pupils have too much fun, too much athletics of the grand-stand type, too many social functions. They often develop the habits of grown-ups, become priggish, domineering, sluggish, and incapable of persistent effort. Lack of home training is responsible for much of this. A real educational crusade must be begun to convince parents of the necessity of careful moral and social training of their children. The increasing agitation against fraternities and athletics is a welcome indication of a sound reaction against the social evils of the high school.

3. High-School Organization.

The fact is that the high school has outgrown its present form of organization. It is sinning every moment against the law that two material bodies can not occupy the same space at the same time. It loses a tremendous number of its pupils (Chicago, 1909: First year, 6,555; fourth year, 1,467). Those who graduate have an education that must be pronounced both unsymmetric and superficial.

(1) *Lack of symmetry.*—This defect is apparent not only in the curriculum as a whole, but is felt in nearly every subject, especially in mathematics, history, and science. The average high-school graduate can at best solve an ordinary quadratic equation or analyze a simple geometric problem. The geometry of solids remains foreign to him, the most helpful trigonometric relations do not form part of his equipment, and, above all, he has not learned to apply his mathematics to the realities of life. This meager outfit, so laboriously acquired, is speedily lost because it is not broad enough to be of real use.

(2) *Lack of thoroughness.*—President Rush Rhees, of the University of Rochester, after a year abroad devoted to a careful inspection of European technical schools, stated at a recent educational meeting that lack of thoroughness is the most widespread defect of the educational work of our American schools. He said that the Amer-

ican educational system gives intellectual power, but fails to impart intellectual life, and that in practical work it fails with reference to methods of study. In mathematics all these defects become accentuated through the tandem system—first algebra, then geometry, then a year of complete interruption, then a review of entirely forgotten principles. A fairly good algebraic foundation may be laid in a year, but geometry most certainly requires more time. A comparison of the percentage of failures in elementary algebra and plane geometry in the tables of the appendix shows that geometry is less successfully taught than algebra, although the poorest pupils drop out at the end of the first year, thus leaving a smaller and more mature body of students.

The effort to make the elective system remedy these defects may be likened to an attempt to cure dyspepsia with liberal doses of olive oil. The digestive apparatus would probably function normally if proper mastication were not rendered impossible by too rapid and ill-arranged feeding. The group system, which is now being substituted for an excessive elective system, will secure greater thoroughness, but not necessarily greater symmetry. Hence, unless the principle of a cosmopolitan high school is to be given up and vocational schools are to take its place a different remedy must be found.

SPECIFIC CAUSES OF FAILURE.

Under this head we must limit ourselves to a brief consideration of the aim, the content, and the methods of secondary mathematics teaching.

Aim.—The one-sided doctrine of mental discipline must go. There need not be any antagonism between theory and practice. Neither Euclid alone nor Perry alone should be our guide. We ought to have a fusion of the abstract and the concrete, a fusion dictated by common sense and free from radicalism in either direction. A teacher who dwells exclusively on half-comprehended, nonproductive subtleties is as much to blame as one who emphasizes merely the How and not the Why. Real applications should be introduced systematically, but the acquisition of a satisfactory technique must not be allowed to suffer. Much of the indifference so often prevailing in mathematical classrooms would disappear if the teachers would take pains to make x and y talk realities and if they poured some life blood into the chimerical formulas of geometry.

Subject matter.—In very many classrooms the textbook or the syllabus of an examining body seems to be the only authority for the content and the relative prominence of the topics considered. Very many pupils fail to get the right point of view. In algebra the whole course must be built around two leading ideas, (1) the

equation as a means of stating and solving numerical relations, (2) the development of the number system. In geometry propositions should be arranged topically, and theorems having small inherent value or not serving as building stones for the whole system should be omitted. The teaching of incommensurables and limits will soon become optional. In all cases the "thought nodes" should stand out prominently in the pupil's mind. The course in secondary mathematics should not be a monotonous array of facts stretching away on an endless wire, but a landscape showing a few towering mountains and many connecting valleys.

Method.—A discussion of method is always dangerous, for here the experienced teacher considers himself on terra firma. It would be the height of conceit, however, not to acknowledge a connection between our poor results and our methods of instruction.

1. In the first place, our school hours should be periods of instruction, of actual thinking and doing, and not merely "recitations." There is no doubt that the "little red schoolhouse," with its insufficient equipment, is responsible for the plan of assigning lessons from a book without previous classroom discussion. In that little school such a practice was a deplorable necessity, since the one teacher in charge was not equal to the task of instructing many classes at once, or of mastering all the prescribed subjects. When the pupils had done their "studying," they were kept busy at the board or were given much written work to do. In this way the textbook and the blackboard gradually usurped the place of the teacher, who soon became a lesson-hearing automaton registering the "marks." That which at first had been a mere makeshift, finally crystallized, under the misunderstood maxims of Rousseau, Fröbel, Jacotot, into a national policy. For this system of teaching was supposed to give full play to the development of individual initiative, of self-activity, etc. Thus it seemed to meet all the requirements of an ideal education. Apparently it turned out self-made men—and it was cheap. How many slow pupils remained submerged forever was not determined. That this outgrowth of the pioneer days of the country should retain so much of its power in the modern high school is a new proof of the overwhelming influence of Anglo-Saxon conservatism. The grotesqueness of the idea becomes apparent when we imagine Socrates assigning a lesson from Homer to Plato or Aristotle.

2. The "recitations" should be less monotonous and less mechanical. The usual procedure consists in giving a brief explanation of the advanced lesson, then sending a large number of pupils to the board, where they consume from five minutes to a whole period. In geometry there is a little more variation. It has been said, to be sure, that mathematics must be written into the mind. But mechanical writing is useless. First the thought, then the symbol. How

little real thinking excessive written work often represents is proved by those pupils who get splendid marks in algebra but are put down as hopeless cases in geometry. The blackboard is a very valuable piece of furniture, but it should be reserved for reviews or snappy drill work. Some teachers have very good success with a system of cards, kept in a filing cabinet. These cards are given out to pupils for miscellaneous blackboard drill.

3. The class, and not the individual, should be the working unit. This is true because individual instruction is usually impossible. Training a class from the beginning to respond in a body makes for greater economy of effort, secures more uniform results, and leaves time for applied work and laboratory methods. This is the real secret of the excellent results obtained in many European schools. Every teacher of mathematics should read Prof. J. W. A. Young's "The Teaching of Mathematics in the Schools of Prussia." Two members of this committee in recent years made a personal comparative study of American and European schools, Mr. Betz in Germany and Miss Wardwell in England. A detailed account of the French system, from the American standpoint, was furnished to the Rochester section of the Association of Teachers in the Middle States and Maryland by Prof. Etzel, of Rochester, the latter having taught more than two decades in French secondary schools. The reports all agree in finding in European schools greater concentration of effort, as well as greater certainty and uniformity of good training, although these results are perhaps obtained at the expense of originality and spontaneity. The first thing that astonishes the American visitor is the small blackboard and the immense amount of oral work. A limited use of the Prussian method can be recommended unconditionally. A number of American teachers have tried it and speak highly of it.

And yet, no method can possibly take the place of a real enthusiastic teacher. The true teacher is an artist. He is ever watching for improvements. He is not dogmatic, but eclectic. The best thoughts of all ages and climes help him to wield an influence which, through his skillful leadership, produces a satisfactory harmony.

REMEDIES.

Throughout this report remedies have been considered in connection with the causes of failure. The remedies suggested in our questionnaire were all received favorably by the teachers except the first, which called for "more thorough teaching by taking more time, e. g., six-year curriculum." Only a small number expressed an opinion in regard to it, thus indicating that the plan is not sufficiently clear.

At the risk of some repetition we propose the following additional remedies:

1. *A six-year curriculum, to begin at the end of the present sixth grade.*—Not a single valid objection can be urged against the plan. The present cramped curriculum seriously interferes with—

(1) The right sort of approach to each subject.

(2) Thoroughness of assimilation.

(3) Permanence of impressions.

(4) Applications of vital interest to pupils.

(5) The coördination and proper sequence of studies.

Many other reasons may be presented why we need more time than formerly. The following list might easily be extended:

(1) We must get away from mechanical textbook teaching.

(2) The prevalence of laboratory methods consumes more time. The spirit of discovery is inconsistent with machine routine.

(3) The old curriculum paid almost no attention to the demands of actual life. It ignored applications.

(4) The requirements of the higher institutions and of the professions are becoming more intensive and extensive.

(5) The complexity of modern life furnishes so many distractions to the young student that more thorough teaching and more reviews are necessary than formerly.

The plan has been tried in some American cities. For a detailed account we must refer to the report of subcommittee 8. The six-year curriculum seems to solve many of our difficulties. All possible objections should be removed by considerations such as the following:

1. It has been tried in Europe. In England, France, and Germany secondary education begins much earlier, usually at the age of 9. This long-continued, unified period of instruction, administered by carefully prepared teachers, is primarily responsible for the supremacy of Europe in science and—perhaps—in foreign commerce. Excellent accounts of the Prussian system may be found in the above-named book of Prof. Young, and in Prof. Klein's Vorträge über den Mathematischen Unterricht an den höheren Schulen (Leipzig, 1907). A brief résumé of present-day teaching of geometry in Europe and America is given in A. W. Stamper's "A History of the Teaching of Elementary Geometry" (Teachers College, Columbia University, 1909). The German high-school boy, in the three types of secondary schools, is given a total of 1,360, 1,680, 1,880 hours, respectively, of unified mathematical instruction. This work is compulsory, and all but the first two or three years of it corresponds to secondary mathemathics in our country; so that 960, 1,280, 1,480 hours, respectively, are given to secondary mathematics. Plane geometry extends over a period of from five to six years; solid geometry over a period

of four years; trigonometry, four years; algebra, six years.[1] Compare with this our less than 200 algebra periods and 200 geometry periods. A pupil who takes four years of mathematics in our high schools gets a maximum of less than 800 periods of instruction, while the German minimum is 960.

2. A six-year curriculum will secure closer contact with the elementary schools.

3. There is no reason why our educational system should not consist of three periods of equal length—primary (6–12), secondary (13–18), higher (19–24).

4. In a more extended secondary course it will be easier to discover a pupil's special aptitudes.

5. The proper psychological moment for teaching certain subjects, e. g., modern languages, can be utilized.

6. An increasing number of pupils find it impossible to complete the present course in four years. Instead of compelling them to fail and drop out, we could adapt the work better to their capacity.

7. The six-year curriculum offers the only hope of overcoming the tandem system. All efforts to secure unified mathematical instruction have been useless under existing conditions.

II. The mathematics teachers of the future must lay a much broader foundation in their own preparation. No candidate lacking a knowledge of the rudiments of trigonometry, analytics, the calculus, and elementary mechanics, ought to be appointed to a high-school position in mathematics. A familiarity with surveying and shop work is also very desirable. Salaries must be made sufficiently high to justify this increased requirement.

III. There must be more expert supervision. This should not be of the nature of petty fault-finding, but should be administered with a spirit of cooperation and inspiration. The experience of other countries justifies the belief that this is a much more effective method of stimulating teachers to better efforts than our periodic examinations, which frequently seem to be narrow and pedantic, and to select incidentals rather than essentials.

CONCLUSION.

It should not be forgotten that many of our educational troubles are fundamentally due to weaknesses in human nature. Hence our aim in this report has been to avoid mere bickering and to retain a broad outlook upon the educational situation in its entirety. Whether we have succeeded in this or not we must leave to the judgment of the reader.

[1] It is of course necessary to remember that ordinarily four weekly periods are devoted to unified mathematics, not to algebra or geometry alone.

APPENDIX.

Reports of Regents Examinations.

Table I.—*Examination of papers.*

Subject.	Papers examined.	Papers passed.	Percentage passed.
1898.			
English, first year	2,561	1,116	43.6
Latin, first year	10,451	5,901	56.5
Elementary algebra	15,166	10,543	69.5
German, first year	5,382	3,936	73.1
Cæsar	4,351	3,264	75.0
Geometry	9,028	4,908	54.4
1899.			
English, first year	3,424	1,554	45.6
Latin, first year	10,293	5,853	56.9
Elementary algebra	14,234	8,784	61.7
German, first year	5,895	4,641	87.7
Cæsar	4,578	3,343	74.8
Geometry	10,112	6,007	59.4
1900.			
English, first year	4,974	2,435	49.0
Latin, first year	11,312	6,351	56.1
Elementary algebra	16,320	10,226	62.7
German, first year	6,582	4,542	69.0
Cæsar	5,198	2,854	54.9
Geometry	9,958	5,048	50.7
1901.			
English, first year	6,484	3,156	48.7
Latin, first year	11,887	6,885	57.9
Elementary algebra	17,085	10,895	63.8
German, first year	6,977	4,536	65.0
Cæsar	6,361	4,610	72.4
Geometry	11,067	6,656	60.1
1902.			
English, first year	8,878	4,549	51.2
Latin, first year	12,185	7,565	62.1
Elementary algebra	18,416	13,380	72.7
German, first year	7,368	5,228	71.0
Cæsar	6,160	4,293	69.7
Geometry	11,030	6,671	60.5
1903.			
English, first year	10,823	5,568	51.5
Latin, first year	11,848	7,899	66.7
Elementary algebra	17,148	10,944	63.8
German, first year	7,146	4,303	60.2
Cæsar	6,807	4,702	69.1
Geometry	11,970	6,941	58.0
1904.			
English, first year	12,957	8,250	63.7
Latin, first year	11,997	7,091	59.1
Elementary algebra	17,773	11,672	65.7
German, first year	7,882	5,506	69.9
Cæsar	7,044	4,880	69.3
Geometry	11,697	7,296	62.4
1905.			
English, first year	15,370	8,737	56.8
Latin, first year	12,490	6,409	51.3
Elementary algebra	18,717	12,005	64.1
German, first year	8,286	5,431	65.5
Cæsar	7,247	4,269	58.9
Geometry	12,576	6,998	55.7

TABLE I.—*Examination of papers*—Continued.

Subject.	Papers examined.	Papers passed.	Percentage passed.
1906.			
English, first year	17,758	10,920	65.1
Latin, first year	11,642	6,776	58.2
Elementary algebra	19,772	14,187	71.7
German, first year	7,473	5,384	72.1
Cæsar	7,277	5,430	74.6
Geometry	11,262	5,825	51.7
1907.[1]			
English, first year	13,186		68.6
Latin, first year	8,509		60.9
Elementary algebra	16,380		84.5
German, first year	4,819		86.1
Cæsar	5,613		79.0
Geometry	7,994		78.8
1908.			
English, first year	21,202		62.3
Latin, first year	13,977		56.0
Elementary algebra	26,426		64.2
German, first year	8,521		84.0
Cæsar	11,620		60.3
Geometry	16,805		62.4
1909.			
English, first year	28,335		80.0
Latin, first year	17,641		62.0
Elementary algebra	31,639		74.4
German, first year	9,716		74.4
Cæsar	12,776		73.1
Geometry	18,562		64.3

[1] In this year transition was made from pass mark 75 per cent to 60 per cent.

TABLE II.—*Summary of pre-academic and academic examinations, 1894–1904.*

	1894	1895	1896	1897	1898	1899
Schools	417	467	517	557	608	639
Papers	371,876	405,557	419,802	445,235	470,471	508,841
Per cent passed	59	60	57	57	61	61

	1900	1901	1902	1903	1904
Schools	672	699	726	730	751
Papers	543,765	538,833	558,301	539,241	564,880
Per cent passed	67	66	69	66	70

TABLE III.—*General averages, in percentages.*

Subjects.	1907	1908	1909
English	70.4	74.4	81.1
German	84.7	75.7	67.5
French	67.5	77.5	68.8
Latin	67.8	55.7 (52,215 papers.)	67.4 (65,726 papers.)
Greek	83.5	68.9	80.4
Mathematics	81.4	66.8 (54,882 papers.)	69.8 (64,400 papers.)
Science	87.0	81.1 (52,907 papers.)	77.9 (56,803 papers.)
History	66.8	72.1	75.7 (35,780 papers.)
Commercial	62.9	69.0	67.8
Business arithmetic	25.3 (408 papers.)	37.8 (1,378 papers.)	28.1 (1,853 papers.)

TABLE V.—*Reports of college entrance examination board.*

Subjects.	1903	1905	1906	1907
English, papers examined	1,857	2,336	2,521	2,996
Passed, 60-100 per cent	72.5	42.7	59.3	58.6
History, papers examined	1,068	1,324	1,370	1,671
Passed, 60-100 per cent	53.2	54.0	47.3	43.2
Latin, papers examined	3,860	5,066	5,693	6,101
Passed, 60-100 per cent	49.4	62.8	49.9	47.9
Cæsar, papers examined	414	598	670	698
Passed, 60-100 per cent	61.4	74.1	61.3	53.6
German, papers examined	964	1,235	1,260	1,629
Passed, 60-100 per cent	68.1	67.6	60.8	75.3
Elementary German	632	778	853	1,082
Passed, 60-100 per cent	68.4	72.6	64.6	64.3
Mathematics, papers examined	3,860	3,000	3,327	3,851
Passed, 60-100 per cent	57.6	48.1	63.9	58.9

TABLE VI.—*Percentage of accepted papers, 1901-1907.*

Ratings.	1901	1902	1903	1904	1905	1906	1907
60-100 per cent	59.3	55.9	58.2	60.1	56.2	55.7	53.3

Conclusions and criticisms.—Concerning the figures of Table VI, Dr. Thomas S. Fiske, the secretary of the board, says:

From an examination of this table it would appear that we must make one or more of the following three inferences—

(1) The question papers set by the board are steadily becoming more difficult.

(2) The board's readers are rating the answer books submitted by candidates with steadily increasing severity.

(3) As the number of candidates examined by the board increases, the quality of the average candidate's preparation is steadily deteriorating.

COMMITTEE NO. IV. MATHEMATICS IN THE PRIVATE SECONDARY SCHOOLS OF THE UNITED STATES.

PLAN OF THE INVESTIGATION.

Membership of the committee.—The following study of the teaching of mathematics in the private secondary schools of the United States has been made by a committee consisting of the chairman and three other members, each of whom has had the assistance of a subcommittee in dealing respectively with boys' schools, girls' schools, and coeducational schools. The field assigned to the committee includes schools connected with religious organizations and preparatory departments of colleges, but not special schools such as trade schools and schools for defectives.

The members of the subcommittee were selected with view to including representatives of various types of schools and, to some extent, of different sections of the country.

Method of collecting the data.—The only publications to which the committee has found it desirable to refer are the annual reports of the United States Commissioner of Education, from which statistics in regard to organization of schools have been taken. At the beginning of the investigation, catalogues of several hundred schools were examined without finding anything of value for the purpose in hand. As the private schools in the United States are subject to no centralized control, no comprehensive view of their organization or of the character of their work could be obtained except by applying to the schools themselves for the information. The method of the questionnaire seemed to be the only one available.

A questionnaire, referred to in the report as the principal questionnaire, was sent, by the United States Commissioner of Education, in behalf of the committee, to all the schools of the country [1] within the committee's field. A supplementary questionnaire covering additional topics was sent by the committee to a limited number of schools selected so as to include different types, sizes, and locations.

[1] The expression "all the schools," used frequently in the report, means all the schools in the records of the United States Commissioner of Education.

Preparation and character of the report.—The returns from the questionnaires were studied by the subcommittees, and reports were prepared by the chairmen. Separate reports upon the fields represented by the subcommittees would involve so much repetition that it has seemed best to combine all the data in a single report, indicating, whenever an important distinction appears, the variation in the practice of the different types of schools.

The greater part of the report is devoted to a statement of the general conditions of mathematics teaching in the private secondary schools of the country. The general plan outlined by the commissioners included, besides a description of present conditions, a statement of progressive movements. This second phase of the investigation was not neglected in planning the questionnaires, but very meager replies were received in answer to requests for criticisms of prevailing methods and recommendations for improvement. The committee has therefore concentrated its attention upon that part of the plan which it understood to be of first importance, and for which reliable data could be secured within the controlling limits of time and expense. The report, then, is for the most part a statement of common-place facts, whose purpose is to give a correct view of the general situation. Obviously it is impossible within reasonable limits of space to give a complete statement even of the facts in hand, but in condensing the data special effort has been made to avoid misleading statements. With this in view the record of the general practice with regard to any feature of mathematics teaching is commonly amplified by a description of the variation from this central tendency.

Of course such a statistical summary of the situation as that described in the preceding paragraph can not reflect the spirit of the work which is being done in the schools. Indeed, the reader may gain the impression that the teaching in the private schools is mechanical and unprogressive. As a partial offset to such an impression a few brief statements of distinctive features of the work in their own schools have been contributed by members of the committee and other directors of mathematics. These statements are quoted in full at the end of the report.

EXTENT AND CHARACTER OF THE DATA.

The principal questionnaire.—The principal questionnaire brought, in time for use in the report, replies from 418 schools, or 23 per cent of all the schools in the country belonging to the field to be studied. The numbers of replies representing different types of schools and the percentage of the total numbers of the schools of the various types are as follows:

Type of school.	Number of replies.	Percentage of all schools of the type.
Preparatory departments of colleges—		
For boys	25	
For girls	20	
For both sexes	47	
Total	92	19
Independent secondary schools—		
For boys	98	35
For girls	94	26
For both sexes	134	20
Total	326	25
Roman Catholic schools	59	16
Other religious schools	94	24
Nonsectarian schools	173	32
In New England [1]	68	33
Middle Atlantic States	93	30
South Atlantic States	42	20
South Central States	28	14
North Central States	78	27
Western States	17	18

[1] The States constituting these various groups are:

New England States: Maine, New Hampshire, Vermont Massachusetts, Rhode Island, Connecticut.

Middle Atlantic States: New York, New Jersey, Pennsylvania.

South Atlantic States: Delaware, Maryland, District of Columbia, Virginia, West Virginia, North Carolina, South Carolina, Georgia, Florida.

South Central States: Kentucky, Tennessee, Alabama, Mississippi, Louisiana, Texas, Arkansas, Oklahoma.

North Central States: Ohio, Indiana, Illinois, Michigan, Wisconsin, Minnesota, Iowa, Missouri, North Dakota, South Dakota, Nebraska, Kansas.

Western States: Montana, Wyoming, Colorado, New Mexico, Arizona, Utah, Nevada, Idaho, Washington, Oregon, California.

The number and percentage of replies with regard to nearly every group of schools referred to would be large enough to give a satisfactory indication of the general character of all the schools of the group, if the schools replying were selected at random. It is natural to suppose that a larger percentage of schools of high grade, than of less efficient schools, would respond to the questionnaire, but it is impossible to say to what extent this supposition is justified. A slight clue to the representative character of the returns may be gained by comparing the size (number of pupils) of the schools reporting with the average size of all the schools. The average enrollment for all the independent private secondary schools for the year 1908–9 is given in the latest published report of the United States Commissioner of Education. As the statistics reported in returns to the questionnaire are for the year 1909–10, the figures are not strictly comparable, but any marked tendency toward the exclusion of the smaller schools would be evident by such a comparison. The respective average enrollments of the reporting schools and of all the schools included in the commissioner's report for 1909 are—for

boys' schools 93 and 82, for girls' schools 62 and 60, and for coeducational schools 91 and 75.

The correspondence between the two sets of figures, taken with the fact that the reported enrollments exhibit a very wide variation, indicates that, while the smaller schools are somewhat less generally represented than the larger ones, a fairly true picture of the general tendency and variability in this one respect may be gained from the reports. The figures in regard to many other details of organization indicate such definite tendencies, with such regular variation toward the extreme values, that they, too, would seem to be fairly representative.

The supplementary questionnaire.—The supplementary questionnaire brought about one-third as many replies as the principal questionnaire. The coeducational schools were poorly represented, and the proportion, as well as number of schools from the North Atlantic States, was much higher than for other sections of the country. The replies include only four Roman Catholic schools and very few preparatory departments of colleges. The proportion of very small schools is much lower than in the returns to the principal questionnaire.

The data obtained from the supplementary questionnaire should not, therefore, be regarded as representative to the same degree as that of the principal questionnaire. The replies to questions of opinion are valuable only as indications of the consensus and variation of opinion among teachers representing chiefly the larger schools in the more populous and wealthy sections of the country.

Following are the numbers of replies from the various groups of schools:

Boys' schools	43
Girls' schools	59
Coeducational schools	37
Total[1]	139
Religious schools	58
Nonsectarian schools	79

Schools situated in—	
New England States	31
Middle Atlantic States	51
South Atlantic States	13
South Central States	11
North Central States	27
Western States	4

Other sources of inaccuracy.—Many replies to both questionnaires were incomplete and some were obviously incorrect. In general, ambiguous answers and those indicating a misunderstanding of the question have been excluded. In the presentation of each topic is given the number or proportion of the replies upon which the statement is based.

In order to reduce the labor of filling out the questionnaire blanks, many questions were so presented that answers could be given by crossing out items from a list of possible replies. Where the items

[1] Two anonymous reports could not be included in the classifications according to location and religious connection.

given are not alternatives, this method is likely to cause error through the failure of the reporting officer to cross out every inappropriate item. Note of this source of inaccuracy is made in the presentation of topics in which it is pertinent.

ORGANIZATION OF PRIVATE SECONDARY SCHOOLS.

Types of schools.[1]—During the Colonial period the American secondary schools were "Latin Grammar Schools," whose function it was to prepare boys for college. During the first half of the nineteenth century the typical secondary school was the academy. It was only incidentally a preparatory school and furnished a broad general course of study. Many of these schools were endowed, and a low tuition fee, or in some cases free tuition, made it possible for children from families in moderate circumstances to attend. Some of the academies were for boys, others—usually called "female seminaries"—for girls, but many were coeducational. They usually provided for boarding students. Many of them were under the control of religious bodies. With the rapid development of the public high schools during the latter half of the nineteenth century, many of the academies ceased to exist or were transformed into public schools. Others became more distinctively preparatory schools.

The private secondary schools of the present day include the following fairly well-marked types: (1) The surviving academies and other more recently organized schools of the same type, including many of the schools connected with religious organizations. (2) Private schools having relatively high tuition fees, usually for one sex. The cities, especially those in the North Atlantic States, usually have day schools of this grade. The boarding schools are ordinarily in the country or in small towns. (3) Preparatory departments of colleges. (4) Secondary departments of elementary schools, including many Roman Catholic schools for girls. There are also a few large, finely equipped coeducational schools maintained in connection with the departments of education of some of the larger universities, or supported by other organizations for the purpose of contributing to progress in education.

Classification of schools according to sex of pupils.—Of the 1,301 independent private secondary schools reporting for the year 1908–9 to the United States Commissioner of Education, 21 per cent were for boys only, 28 per cent for girls only, and 51 per cent were for both sexes. Of 422 preparatory departments of colleges (not including those under State control), 21 per cent were for boys, the same percentage for girls, and 58 per cent were coeducational.

[1] The first paragraph is based upon a monograph on Secondary Education by Elmer Ellsworth Brown in a series of monographs on Education in the United States, edited by Nicholas Murray Butler, Albany, 1900. The second paragraph is based upon the author's observation, combined with returns from the questionnaires.

Geographical distribution.—The Middle Atlantic and the North Central sections have each about 300 of the independent schools. New England, the South Atlantic, and the South Central sections have each about 200, and nearly 100 schools are in the Western States. The proportion of boys' schools is relatively high in the eastern part of the country, and that of the girls' schools in the North Central and Western States. About half of the schools in each section of the country are coeducational, except in the South Central and Western States, where the coeducational schools form, respectively, 65 per cent and 34 per cent of the total. Practically none of the New England colleges have preparatory departments, and there are comparatively few in the middle Atlantic States. In the other sections of the country, however, the colleges commonly have such departments, and in the North Central States alone there are 180 of them. A majority of these preparatory departments are coeducational, but in the Southern States there are many connected with colleges for women.

Religious connection.[1]—Twenty-nine per cent of the independent private secondary schools are controlled by the Roman Catholic Church, and 30 per cent are connected with other religious denominations. Thirteen per cent of the preparatory departments of colleges are Roman Catholic and 69 per cent are connected with other churches. Two-thirds of the schools in New England are nonsectarian, while in other sections of the country the majority are religious schools. The Roman Catholic schools for girls are very numerous in the Central and Western sections, forming 70 per cent of all the girls' schools in that region. A majority of the Roman Catholic schools for boys are connected with colleges; 50 of the 88 preparatory departments for boys are Roman Catholic. Sixty per cent of the independent schools and 70 per cent of the preparatory departments connected with other religious denominations are coeducational.

Day schools and boarding schools.—Of 404 schools reporting on this point in answer to the principal questionnaire, 31 per cent are day schools, 10 per cent boarding schools, and 59 per cent take both day and boarding pupils. The same order holds in all three classes of schools (boys', girls', and coeducational) and in all sections of the country; that is, the greatest number take both day and boarding students, and the schools for boarders only are least numerous. There are very few schools for day pupils only in the South and West.

Age of schools.—Three hundred and eighty-eight schools replying to the principal questionnaire gave the date of establishment (or that of the secondary department). (See fig. 1.) The ages range from 1 year to 265 years, a few of the oldest preserving the name "Grammar School." Twenty-one of the schools are over 100 years old, includ-

[1] It is safe to say that many schools reported as religious schools are subject to no control by religious organizations. We have no means of separating these schools from those properly classified as religious institutions.

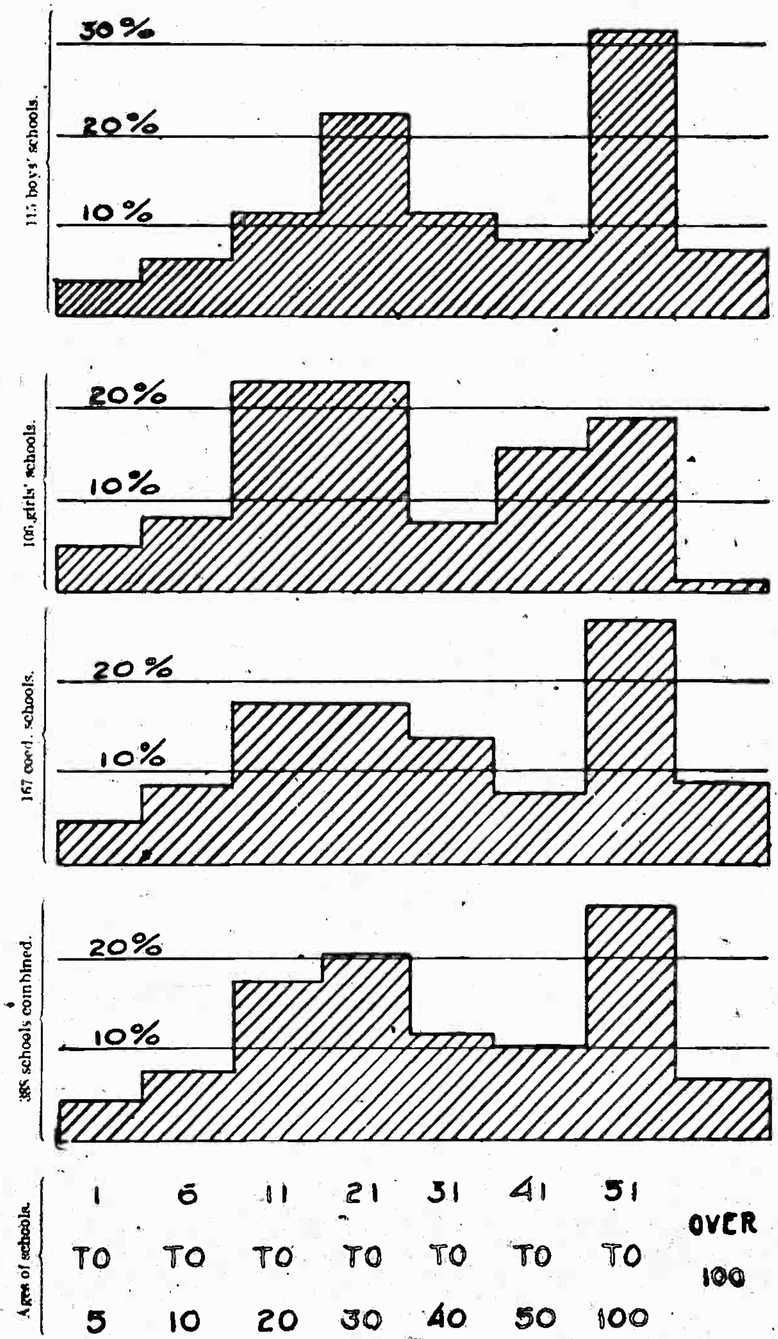

FIG. 1.—Ages of schools; percentage distribution.

ing some of the oldest academies. Only 16 have been established within 5 years. The median age of the boys' schools lies between 40 years and 50 years, that of the coeducational schools between 30 and 40, and that of the girls' schools between 20 and 30. Thirty-seven per cent of the boys' schools and 34 per cent of the coeducational schools are over 50 years old, while only 19 per cent of the girls' schools are as old as this.

Number of pupils.—In 1908–9[1] the average enrollment in the independent private secondary schools was, in boys' schools 82, in girls' schools 60, and in coeducational schools 36 boys and 39 girls. The variability in the size of schools may be judged from the following table based on 345 replies, although the average enrollment in these schools is about 18 per cent higher than that of all the schools referred to above. (See fig. 2.)

Distribution of 345 schools, including preparatory departments of colleges, according to number of pupils.

Schools.	Enrollment.							Total number of replies.
	More than 500	401 to 500	301 to 400	201 to 300	101 to 200	51 to 100	50 or less.	
Boys' schools		1	3	7	24	30	36	101
Girls' schools				1	11	23	45	80
Coeducational schools	2	1	6	12	44	33	66	164
All schools	2	2	9	20	79	86	147	345

The two largest schools, which have respectively 1,048 and 580 pupils, are both coeducational day schools and are connected with higher institutions, situated in one of the largest cities in the country. The largest boys' school, with an enrollment of 500, is an independent secondary school—one of the old academies—located in a small town in New England and taking both day and boarding students. The largest girls' school reporting its enrollment has 216 pupils. It is an independent day school in a New England city of moderate size.

It will be noticed that more than half of the girls' schools and 42 per cent of all schools had not more than 50 pupils each. Eighty-five per cent of the girls' schools had not more than 100 pupils each, and in 90 per cent of all the schools the enrollment was not over 200.

Number of pupils per teacher.—In 1905–6[2] the average number of pupils per teacher in all the independent private secondary schools was as follows: For girls' schools 6.9, for boys' schools 10.4, and for coeducational schools 14. The variability of the number of pupils to one teacher for 342 schools[3] is shown below.

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[1] Report of United States Commissioner of Education, 1909.

[2] Report of the United States Commissioner of Education, 1906. This is the most recent report giving detailed statistics of secondary schools.

[3] Data of principal questionnaire.

FIG. 2.—Sizes of schools; percentage distribution.

Number of pupils per teacher.

Schools.	46 to 50	41 to 45	36 to 40	31 to 35	26 to 30	21 to 25	16 to 20	11 to 15	6 to 10	5 or less.	Total number of reports.
Number of boys' schools					8	6	10	32	42	12	105
Number of girls' schools						2	7	15	37	25	86
Number of coeducational schools	2	3	2	2	5	15	27	57	31	7	151

Eighty-two per cent of the boys' schools and 90 per cent of the girls' schools have not more than 15 pupils per teacher, while only 63 per cent of the coeducational schools come within this limit. The nine coeducational schools in which the ratio is higher than 30 pupils to one teacher are nearly all in the Southern States.

Length of school year.—The length of the school year as reported by 407 schools is as follows:

With the exception of one school which has only a single year of secondary work, the minimum school year was 25 weeks in a small coeducational day school in New York City. The maximum reported by three schools was 42 weeks. Sixty-four per cent of the boys' schools, 50 per cent of the girls' schools, and 87 per cent of the coeducational schools have a year of from 36 to 40 weeks. All but 13 out of 407 schools have more than 30 weeks. Most of the schools with a very short year are girls' schools in the East. The Roman Catholic schools commonly have 40 weeks. The school year is shorter in the East than in the Central and Western sections; 41 per cent of the eastern schools have a year of 35 weeks or less, while in the rest of the country only 15 per cent of the schools reporting have so short a year.

Connection with other departments.—Among the 418 schools replying to the questionnaire, elementary departments are maintained by 60 per cent of the boys' schools, 79 per cent of the girls' schools, and 44 per cent of the coeducational schools. In boys' schools the elementary department is as a rule relatively small, and in girls' schools and coeducational schools the same is true in a majority of cases, but many of the Roman Catholic schools for girls and for both sexes have large elementary departments, with comparatively few secondary students. As already stated, 22 per cent of the schools replying are connected with colleges. Besides these, about one-sixth of the schools give some work of collegiate grade.

Courses of study.—Practically all the schools offer college preparatory courses. About 20 per cent of all the schools offer only a single course—usually planned to meet college requirements—although some choice of studies within this course is often allowed. This practice (single course) is commonest in boys' schools and least common in girls' schools. A more general plan is to offer two courses, a

college preparatory course and either a scientific course, a general course, or a commercial course. Many schools differentiate more widely, offering three, four, or even five courses. In 1908–9,[1] 29 per cent of the boys and 10 per cent of the girls enrolled in independent private secondary schools were preparing for college. A majority (53 per cent) of these boys were in scientific courses, while 71 per cent of the girls were in classical courses. Sixty-one per cent of the boys and 23 per cent of the girls in the graduating classes intended to enter college. Commercial courses are given in comparatively few (about one-fifth) of the girls' schools. Most of the Roman Catholic schools for boys give commercial courses, and a relatively large proportion of the Roman Catholic schools for girls give such courses.

Except for commercial students, four years is ordinarily the length of the secondary course, pupils entering at about 14 years of age. About an equal number of schools—7 or 8 per cent of all—have courses of three years and five years, respectively, and a few schools make six years the normal period. The schools whose courses are longer than four years usually admit pupils at an earlier age than the others, or give work in advance of the requirements for admission to college. Among the schools which maintain elementary departments there may be little real distinction between those which report a secondary course of six years and those which label only four of those years "secondary." Commercial courses vary in length from one to four years. In the boys' schools four years is the common length, and in girls' schools two years, while in the coeducational schools courses of four years and two years are about equally common.

Number and sex of teachers.—In 1905–6[2] the average number of teachers per school was, for boys' schools 7.2, for girls' schools 7.8, and for coeducational schools 5.1. In each case the average is smallest for the South Central States. It is largest for the Middle Atlantic States, except in coeducational schools, in which the Western States give the highest average. The variability in the number of teachers per school, among 327 schools,[3] is as follows:

Percentage distribution of 327 schools according to number of teachers.

	Number of teachers. More than 20.	16 to 20.	11 to 15.	6 to 10.	5 or less.	Total number of schools included.
Percentage of boys' schools	6	5	8	53	28	89
Percentage of girls' schools	1	5	19	39	36	65
Percentage of coeducational schools	3	5	9	36	47	153

[1] Report of United States Commissioner of Education. 1909.
[2] Ibid., 1906.
[3] Data of principal questionnaire

The maximum number of teachers reported for the secondary department of any one school was 21 for girls' schools, 38 for boys' schools, and 54 for coeducational schools. Several of the coeducational schools report only one teacher. Eight per cent of all the schools reporting have more than 15; 80 per cent have not more than 10.

In 1905-6,[1] 93 per cent of the teachers in boys' schools were men; 93 per cent in girls' schools were women; while in the coeducational schools 54 per cent were women.

THE DEPARTMENT OF MATHEMATICS.

Number and sex of mathematics teachers.—In the replies to the principal questionnaire, about 90 per cent of the boys' schools and coeducational schools and 75 per cent of the girls' schools gave lists of their teachers of mathematics with the courses taught by each teacher. The records of about 800 teachers are included in these reports.

In the boys' schools practically all the classes in mathematics are taught by men, in the girls' schools 93 per cent of the teachers are women, while in the coeducational schools 63 per cent are men.

In about one-third of the schools reporting, the work in secondary mathematics is done by a single teacher, who, as often as not, teaches other subjects also. Several of the large boys' schools have as many as eight teachers giving all their time to mathematics.

Teachers' assignments.—About 30 per cent of the teachers teach only mathematics and give at least two hours a day to teaching. About 60 per cent are reported as teaching other subjects also. In the remaining cases, only mathematical courses are listed, but, as only one or two courses are assigned to each teacher, it is probable that many of these teachers have other teaching assignments which were not recorded. In the Roman Catholic schools it is the common practice to divide the work in mathematics among several teachers, each of whom teaches several other subjects.

In the list of subjects other than mathematics assigned to mathematics teachers, every subject in the curriculum appears. Those most frequently mentioned are, in order: Natural science (especially physics), Latin, modern languages, and commercial subjects.

About one-eighth of the teachers have classes in mathematics in elementary or collegiate departments as well as work of secondary grade.

The returns giving the amounts of time required of teachers in teaching and in other school duties are not sufficiently reliable to warrant a quantitative statement. They indicate a very wide variation, some teachers giving only a single course while others are reported as teaching an incredible number of hours. As illustrations

[1] Report of United States Commissioner of Education, 1906.

of practice, in some of the well-known schools, whose reports were evidently made with care, the following figures are given, but it is not intended to imply that they are typical:

Schools.	Number of minutes of teaching.	Number of minutes of other duties.
1. A boys' boarding school in Pennsylvania	675 to 950 (average, 876).	400 to 1,150 (average, 690).
2. A boys' day school in New York City	1,200 to 1,400 (average, 1,270).	250 to 460 (average, 390).
3. A girls' boarding school in California	705	460.
4. A girls' day school in Massachusetts	640 to 960 (average, 800).	465 to 785 (average, 625).
5. A coeducational boarding school in Ohio	1,200 to 1,380 (average, 1,290).	None required.
6. A coeducational day school in Chicago, Ill.	1,000 to 1,250 (average, 1,050).	None.

Teachers' salaries.—About 60 per cent of the replies from boys' schools and coeducational schools reported facts in regard to the salaries of mathematics teachers doing secondary work. Only 44 per cent of the girls' schools reported salaries. The reports from Roman Catholic schools, with very few exceptions, stated that their teachers receive no salaries, but are supported by the religious orders to which they belong.

The questionnaire called for maximum, minimum, and average salaries in order to secure data on the variation within the schools which have several teachers. In some cases the maximum only was given, in some the minimum, and in others the average. Some of the replies showed that the question was understood to call for variation in a single teacher's salary according to the schedule of salaries in force in the school.

The figures, therefore, are probably not as reliable as most of the data presented. They will serve, however, to indicate in a general way the distribution of salaries in different types of schools and in different sections of the country. In the following figures the salaries of principals who are also teachers of mathematics, and those of teachers receiving board as a part of their remuneration, have been excluded. (See fig. 3.)

In the boys' schools, the salaries reported vary from $50 per month to $2,700 per year. Ten schools reported a maximum salary of $2,000 or over. Of the 39 schools reporting an average salary or the salary of a single teacher, 25 reported sums lying between $600 and $1,200, and the rest, with one exception, between $1,300 and $2,000.

In the girls' schools, the lowest salary reported was $40 per month. Only one school—a day school in New York City—gave a sum higher than $1,200. In this the maximum is $2,000. Sixteen of the 30 schools which report an average salary place this between $700 and $1,000.

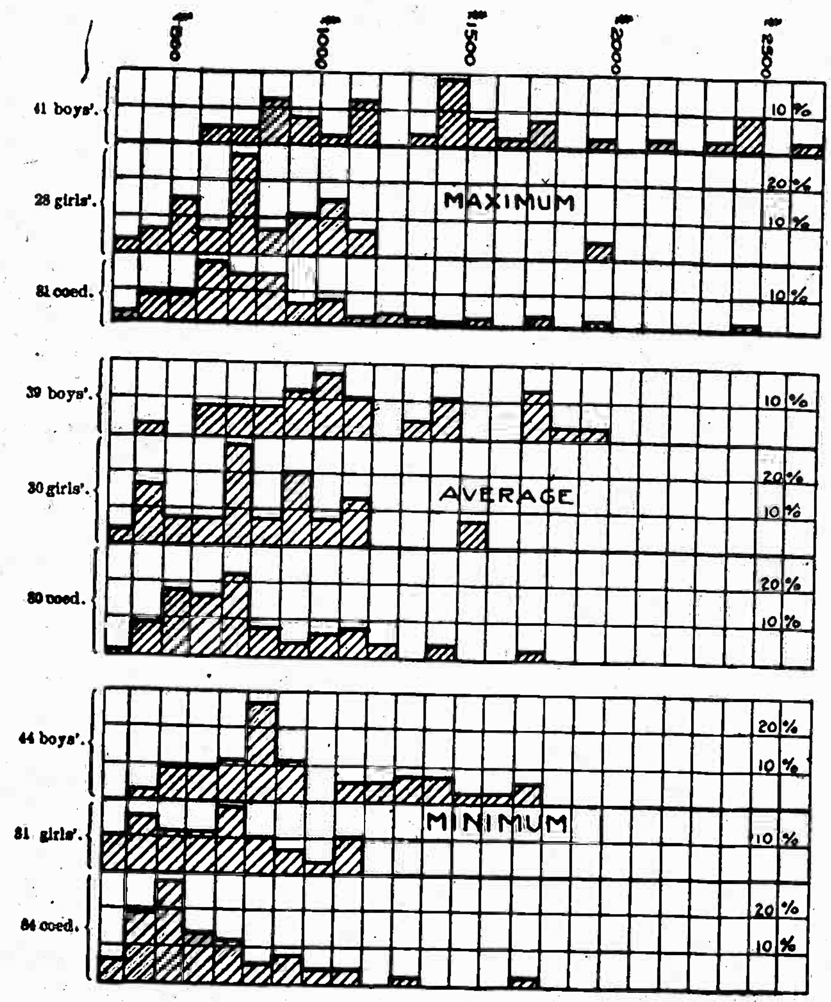

FIG. 3.—Percentage distribution of salaries of mathematics teachers.

In the coeducational schools the figures range from $35 per month to $2,500 per year. Besides the one reporting this maximum, only one school has a maximum as high as $2,000. Ten have maximum salaries ranging from $1,200 to $1,800. Eighty coeducational schools gave an average salary. In more than half of these the sum varies from $500 to $800. In 20 of the 80 schools it lies between $800 and $1,200.

The average of the salaries reported as the salary of a single teacher or the average of the salaries of all mathematics teachers in a school is, for boys' schools between $1,100 and $1,200, for girls' schools between $700 and $800, and for coeducational schools practically the same as for girls' schools.

On the whole, the salaries reported from schools of the North Atlantic States are considerably higher than in other parts of the country, although the few schools representing the far West give figures closely corresponding to those in the North Atlantic States.

Academic and professional training of teachers.—The supplementary questionnaire called for data in regard to the training and professional experience of teachers of mathematics. The replies represent 146 schools with 282 teachers. As one would expect, the teachers referred to in the reports from boys' schools are practically all men and from the girls' schools women. There is a much larger proportion of male teachers in the coeducational schools reporting than in coeducational schools generally.

The following table gives a distribution of the teachers for whom data were reported according to total length of experience in teaching and also according to length of service as teachers of mathematics. (See fig. 4.)

Years of experience in teaching.	Distribution of teachers by years of experience in teaching.			Distribution of teachers by years of experience in teaching mathematics.		
	Boys' schools.	Girls' schools.	Coeducational schools.	Boys' schools.	Girls' schools.	Coeducational schools.
1 to 5 years	15	24	32	25	34	35
6 to 10 years	26	20	18	26	20	18
11 to 15 years	24	17	11	18	10	10
16 to 20 years	8	15	5	8	17	8
21 to 25 years	8	10	4	8	6	2
Over 25 years	13	7	6	13	3	4
Total	94	93	76	98	90	77
Average	14.0	12.6	9.7	12.8	10.5	9.1

It appears that 35 per cent of these teachers have taught mathematics not more than 5 years and nearly 60 per cent not more than 10 years. The average length of service is greatest in the boys' schools and least in the coeducational schools.

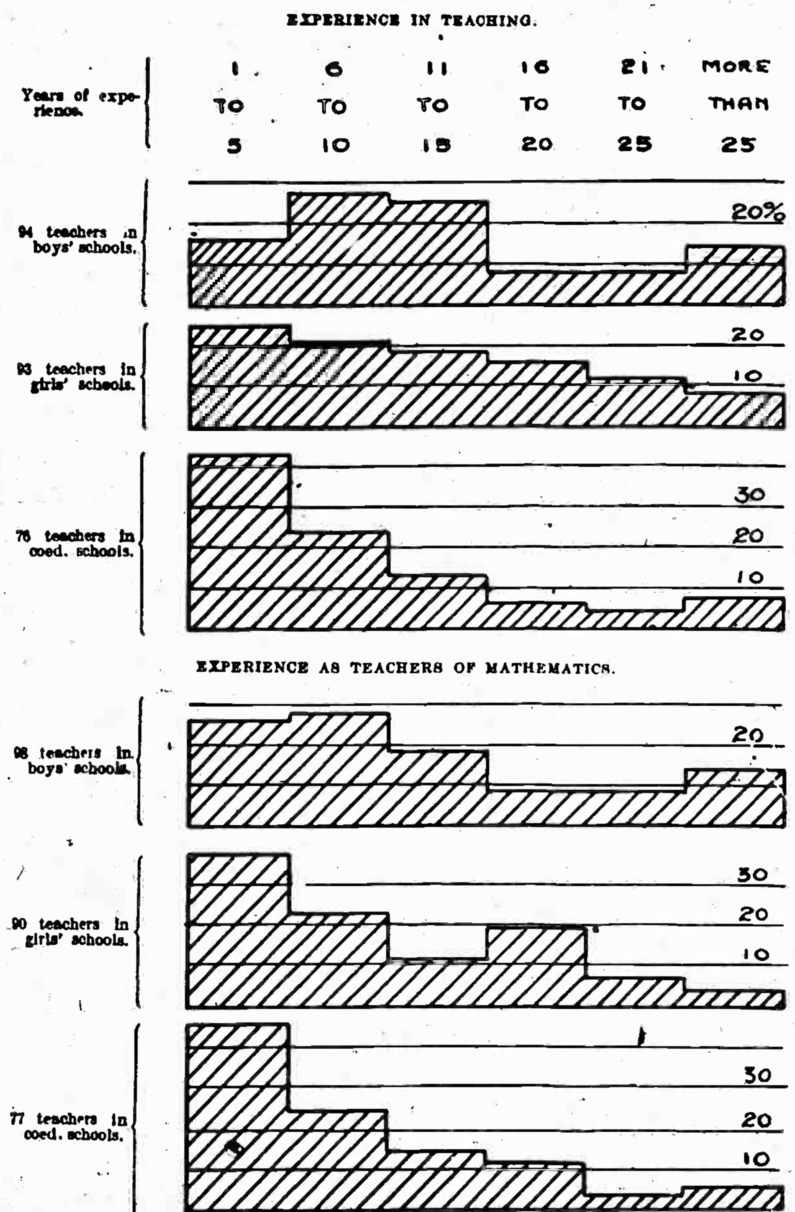

Fig. 4.—Length of teaching experience of mathematics teachers; percentage distribution.

The facts relating to the academic and professional education of the teachers reported are, in brief, as follows:

Academic education.	Boys' schools.	Girls' schools.	Coeducational schools.
Number of teachers reporting	101	92	79
Number having doctor's degree	3	0	2
Number having master's degree (no higher)	22	12	21
Number having bachelor's degree (no higher)	58	53	46
Total number holding degrees	83	65	69
Number having partial college education (not including summer school work)	9	6	3
Number having full normal course	2	3	0
Number having partial normal course		5	2
Number having full high-school course	3	4	3
Number having partial high-school course	1	4	1
Number mentioning no academic training	3	5	1
Total number not holding degrees	18	27	10
Percentage not holding degrees	18	29	13

Eighty per cent of all the teachers for whom the data were given were holders of degrees. The proportion is not as high (about 70 per cent) for teachers in girls' schools as for those in schools for boys and for both sexes, and a smaller portion of degree holders teaching in the girls' schools have taken a second degree than is the case among teachers in the other institutions.

About half of the holders of degrees report academic or professional training in addition to that required for a degree, the proportion being highest in schools for girls. Of the teachers in girls' schools who have received no degree, nearly all have had some definite professional education or academic work of collegiate grade.

Seventeen of the male teachers have had training for other professions: Four for law, two theology, one medicine, one agriculture, and nine engineering.

Direction of the department of mathematics.—In a small majority of the schools replying to the principal questionnaire, the work in mathematics is directed by the principal, who, more often than not, teaches some of the classes in mathematics. In Roman Catholic schools the principal usually directs the work and does not often teach mathematics.

In the other schools one of the teachers of mathematics usually acts as director, but in the preparatory departments of colleges one of the college teachers frequently supervises the preparatory work. Usually, but not always, he teaches some of the classes in secondary mathematics.

In the boys' schools the directors are men in all the schools reporting; in the girls' schools one-sixth of the directors are men, while in the coeducational schools seven-eighths are men.

Size of classes.—About two-thirds of the schools replying to the principal questionnaire gave the numbers of pupils in recitation sections in mathematics. These represent a very wide variation, a few classes having 60 to 70 pupils each, while many have less than five. In general the size of the school determines the size of the classes, comparatively few schools (about one-fifth of the boys' schools and coeducational schools and a smaller proportion of the girls' schools) having more than one section of a given class.

In schools having parallel sections the variation corresponds closely to that for all the schools, excepting at the extremes of the scale. Few of these parallel sections have less than 10 or more than 30 pupils. A comparatively large proportion of the girls' schools have very small divisions, even when classes are divided. The largest schools (for boys and for both sexes), which have in some cases as many as eight parallel sections, organize recitation sections of about 25 pupils each.

The replies in which enrollments were clearly recorded included over 1,500 recitation sections, about 500 in boys' schools, 300 in girls' schools, and 700 in coeducational schools. (See fig. 5.) On the whole the size of classes is about the same in the boys' schools and coeducational schools (average between 16 and 17) and considerably smaller in the girls' schools (average 11). Nearly 30 per cent of the sections from girls' schools have from 1 to 5 pupils each, while only 12 per cent of the sections from other schools have as small an enrollment as this. About 80 per cent of the sections from girls' schools have not more than 15 pupils each. About half of the other sections fall within this limit. From 85 to 90 per cent of all the divisions have not more than 25 pupils each.

Time allotment.[1]—Intelligible answers to the question in regard to the time allotment were given in about 80 per cent of the replies. The normal plan for mathematical courses is to have five recitations per week but four are not infrequently assigned, and some of the boys' boarding schools have six. In the North Atlantic States some schools have only three recitations per week.

The tendency is to reduce the number of periods in the later years of the course. In the last year, where elective courses are common, there is much variation and in many cases it is possible for a student's program to include seven or more periods in mathematical subjects. In scientific courses seven periods per week are sometimes required in the last year.

The length of the recitation period varies from 20 minutes to 75 minutes, but 40 or 45 minutes is the usual length.

[1] Data of principal questionnaire.

Grading and promotion of pupils.[1]—In regard to the admission of pupils to the school, 41 schools use no examinations but admit pupils on trial, on credentials from other schools, or—as in nearly all cases—on both conditions. Thirty-one schools use an oral examination in connection with credentials or trial or both, 4 use a written examination as the sole basis for admission, and 59 use written examinations

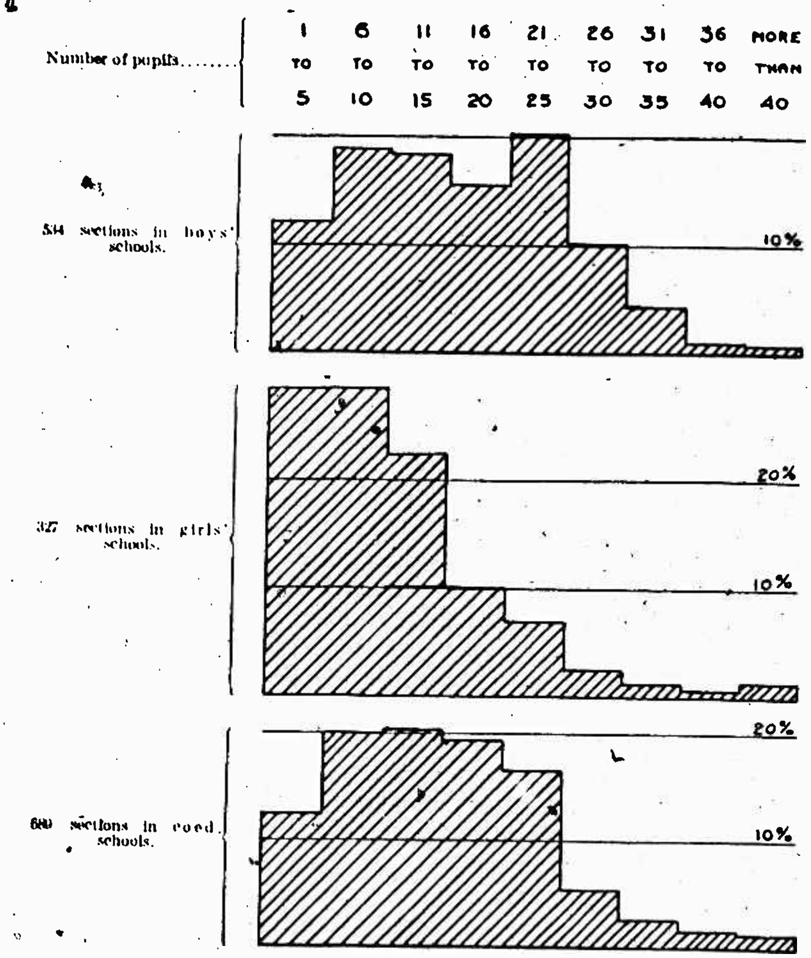

FIG. 5.—Sizes of mathematics classes; percentage distribution.

in connection with one or more of the other methods referred to. In 90 per cent of the schools some member of the mathematics department is consulted in regard to the grading of new pupils.

Promotion in mathematics is independent of the standing in other subjects in 80 per cent of the schools reporting (about 100). In a

[1] Data of the supplementary questionnaire.

few schools a pupil is not promoted in any subject unless he passes in all; in some cases promotion is based upon the pupil's average standing in all subjects; in others a pupil is obliged to repeat mathematics even though he has done well in it, if he fails in a certain proportion of his other work. Several say that standing in mathematics and English or mathematics and Latin determines promotion in all subjects.

No definite statements are made in regard to the bearing of these plans of promotion upon the effectiveness of the teaching in mathematical classes, but so far as this testimony goes, there would appear to be few instances in which teachers of mathematics are hampered by unsatisfactory grading of pupils. Some data in regard to examinations for promotion are given later in the report.

THE COURSE OF STUDY.

Determination of the content of the course of study.—In answer to the supplementary questionnaire about 100 schools gave information in regard to the control of the course of study. In 10 cases the content of the course is said to be determined by college entrance requirements, and in 5 cases by trustees, governing board, or some outside authority. In two-thirds of the other schools the principal takes some part in the determination of the course, usually consulting with teachers, head of department, faculty, or trustees. A head of department acts alone in 14 schools in dealing with the content of the course. In 9 schools teachers of mathematics are free to lay out their courses of study, and in 28 others they are consulted. The plan mentioned by the greatest number of schools (20) calls for joint action by the principal and the teachers of mathematics.

A large majority of the schools report that the director of mathematics is free to modify the course of study to suit the needs of particular classes, but the approval of the principal is frequently mentioned as a condition. College requirements are often referred to as a definite limitation upon modification of the course, and the details reported in answer to the principal questionnaire would seem to indicate that these requirements have a controlling influence in most schools. So far as one can judge from the meager comments on "flexibility of the course" the freedom of the teachers is usually limited to order of topics, proportion of time devoted to different topics, and methods of teaching. There is nothing in the replies to show that courses are shortened or changed in content to suit the characteristics of particular classes.

Teachers are commonly free to select their own textbooks, although the approval of the principal or head of department is often required. In 14 schools books are selected by the principal and in 17 by the head of department.

The course in general.—Eighty per cent of the returns from the principal questionnaire report the mathematical subjects in each year of the course. Most of the schools report no differentiation for students anticipating different vocations, but schools giving commercial courses commonly require commercial arithmetic—sometimes in place of geometry—for pupils of this department. In many schools separate sections are organized for college preparatory students, and in some cases, particularly in the girls' schools, such students are required to take more work in mathematics than others.

With very few exceptions all students are required to study elementary algebra and plane geometry. Solid geometry is given in about 80 per cent of the boys' schools, 40 per cent of the girls' schools, and 65 per cent of the coeducational schools. Plane trigonometry is given in about 75 per cent of the boys' schools, 35 per cent of the coeducational schools, and 18 per cent of the girls' schools. "Advanced" algebra is given in nearly half of the boys' schools, rarely in the others. Solid geometry, plane trigonometry, and advanced algebra are frequently elective subjects, except for students of the scientific course. They are seldom given in the girls' schools of the North Atlantic States. A very few schools give courses in spherical trigonometry, analytical geometry, and calculus.

Order of subjects.—The most general arrangement of subjects in the four years' course is that which places algebra in the first two years, plane geometry in the third, and solid geometry and plane trigonometry in the fourth. There are, however, many variations from this plan.

Arithmetic, either alone or combined with algebra, is given in the first year in about one-fourth of the boys' schools and coeducational schools. It is given less frequently in the girls' schools.

Plane geometry is frequently placed in the second year instead of algebra. In many of the boys' schools the time assigned to mathematics in the second and third years is divided between algebra and geometry, but this practice is much less common in the other schools.

Solid geometry is sometimes given in the third year, usually as a continuation of the course in plane geometry, which occupies the first part of the year. This plan is common among the coeducational schools, about one-fourth of which give no mathematics in the fourth year.

The final year of the course is characterized by much variation. Several schools for girls and for both sexes give courses in practical arithmetic. Some schools defer plane geometry until this year and others give a second course in elementary algebra or a combined course in algebra and plane geometry. About one-third of the girls' schools report courses for review of algebra and plane geometry, which are usually required for college preparatory students.

Schools which give solid geometry, trigonometry, and advanced algebra usually assign these subjects to the final year.

Content of courses in the various subjects.—The principal questionnaire called for a detailed description of the course of study by references to textbooks, with comments indicating departures from the order or scope of the books referred to. This part of the questionnaire brought the least satisfactory returns. In many cases no information was given, and in others it was obviously incomplete. About 40 per cent of the returns, representing about 160 schools, indicated the scope of the courses. As these conform closely to college-entrance requirements, perhaps the most satisfactory method of describing the content of the course will be to quote the definitions of the College Entrance Examination Board, and to indicate roughly any tendencies to depart from these definitions. They are as follows:

a. Elementary Algebra—Algebra to Quadratics and Quadratics and beyond.

(a. i. Algebra to Quadratics.)

The four fundamental operations for rational algebraic expressions.

Factoring, determination of highest common factor and lowest common multiple by factoring.

Fractions, including complex fractions, and ratio and proportion.

Linear equations, both numerical and literal, containing one or more unknown quantities.

Problems depending on linear equations.

Radicals, including the extraction of the square root of polynomials and of numbers.

Exponents, including the fractional and negative.

(a. ii. Quadratics and beyond.)

Quadratic equations, both numerical and literal.

Simple cases of equations with one or more unknown quantities that can be solved by the methods of linear or quadratic equations.

Problems depending on quadratic equations.

The binomial theorem for positive integral exponents.

The formulas for the nth term and the sum of the terms of arithmetical and geometric progressions, with applications.

It is assumed that pupils will be required throughout the course to solve numerous problems which involve putting questions into equations. Some of these problems should be chosen from mensuration, from physics, and from commercial life. The use of graphical methods and illustrations, particularly in connection with the solution of equations, is also expected.

b. Advanced algebra.

Permutations and combinations, limited to simple cases.

Complex numbers, with graphical representation of sums and differences.

Determinants, chiefly of the second, third, and fourth orders, including the use of minors and the solution of linear equations.

Numerical equations of higher degree, and so much of the theory of equations, with graphical methods, as is necessary for their treatment, including Descartes' rule of signs and Horner's method, but not Sturm's functions or multiple roots.

c. Plane geometry.

The usual theorems and constructions of good textbooks, including the general properties of plane rectilinear figures; the circle and the measurement of angles; similar polygons; areas; regular polygons, and the measurement of the circle.

The solution of numerous original exercises, including loci problems.

Applications to the mensuration of lines and plane surfaces.

d. Solid geometry.

The usual theorems and constructions of good textbooks, including the relations of planes and lines in space; the properties and measurement of prisms, pyramids, cylinders, and cones; the sphere and the spherical triangle.

The solution of numerous original exercises, including loci problems.

Applications to the mensuration of surfaces and solids.

e. Trigonometry.

Definitions and relations of the six trigonometric functions as ratios; circular measurement of angles.

Proofs of principal formulas, in particular for the sine, cosine, and tangent of the sum and the difference of two angles, of the double angle and the half angle, the product expressions for the sum or the difference of two sines or of two cosines, etc.; the transformation of trigonometric expressions by means of these formulas.

Solution of trigonometric equations of a simple character.

Theory and use of logarithms (without the introduction of work involving infinite series).

The solution of right and oblique triangles and practical applications, including the solution of right spherical triangles.

f. Plane trigonometry.

The subject is the same as the preceding except that no topics from spherical trigonometry are included.

In algebra many of the schools reporting limit the work of the first year to the fundamental operations with integral and fractional expressions, factoring, and the solution of linear equations with one or more unknowns. Others cover all the topics included in the foregoing definition of "Algebra to Quadratics," and a few give still more advanced work. The complete course in algebra as reported by some schools is more comprehensive than the definitions, including some or all of the following topics: Inequalities, imaginaries, variation, harmonical progressions, infinite series, undetermined coefficients, logarithms, probability, continued fractions, summation of series, exponential and logarithmic series, theory of numbers.

In geometry and plane trigonometry the scope of the courses is more nearly uniform than in algebra. Nearly all the schools reporting follow the textbooks closely, almost the only change mentioned being the omission by many schools of theorems on maxima and minima of plane figures. In solid geometry one widely used textbook adds a chapter on conic sections to the topics included in the definition of the College Entrance Examination Board, but many schools omit this.

METHODS.

General classroom methods.—In the schools represented by the returns from the supplementary questionnaire a part of nearly every lesson is devoted to oral recitation of the assigned lesson, but this plan is used less frequently in algebra than in geometry. In many of the schools oral drill occupies a small part of each period, and in most of them blackboard work by pupils is called for regularly. A common practice is to have half the class at the board while the other half works at seats or recites the lesson of the day. In most schools classes are required occasionally to answer in writing questions on the assigned lesson. A new topic is usually developed in class before home work upon it is assigned.

Preparation of lessons.[1]—The younger pupils and those whose work is deficient are in many schools required to prepare their lessons under the supervision of teachers. The older pupils in good standing generally study in their rooms, if in a boarding school, or at home, if day pupils. A few schools report a regular weekly period in which the teacher trains the pupils in methods of preparing a new lesson.

The lesson to be prepared out of class, in algebra, consists usually of problems taken from the textbook. In geometry the lesson commonly calls for a studying of theorems to be recited in class. Other types of lessons reported are: Study of definitions, rules, methods of solution, and general principles; writing of notes, outlines and summaries, demonstration of original theorems, solution of numerical problems based on geometrical relationships, problems in geometrical construction, practical problems, invention of problems to illustrate a principle, plotting of graphs, construction of models, and out-door measurements.

Special methods and devices.—The following specific methods and devices were listed in the principal questionnaire, and reporting officers were asked to cross out those not in use, to indicate the time during which each method had been used, and to state whether it were still in use. The number after each item indicates the percentage of replies in which the method was reported to be in use.

Use of squared paper	49
Preliminary course in observational geometry preceding formal demonstration	32
Laboratory method	27
Use of historical material	26
Heuristic method	24
Development of course in geometry without a textbook	20
Paper folding	19
Study of logic in connection with geometry	16
Mathematical recreations	10
Organization of a mathematics club among the pupils	4

[1] Based on returns from the supplementary questionnaire.

In nearly all cases methods once used were reported to be still in use, but a few schools after a trial have abandoned the teaching of geometry without a textbook, and a few others the use of a preliminary course in observational geometry.

Perhaps the most noteworthy feature of this record is the general use of squared paper, which has been introduced within five or six years by most of the schools which stated the period of use.

Source and variety of problems.—The most common practice is to use the problems in the pupil's textbook or to supplement these by dictated problems, but in many schools the pupils have at hand more than one collection of exercises. Historical problems, and those drawn from other school subjects and from current events, are said to be used in about half of the schools replying. Problems based on industrial appliances and processes and those involving out-of-door measurements are used in many of the boys' schools and coeducational schools.

Equipment.—Three-fourths of the returns to the principal questionnaire indicate, on lists of apparatus and tools, which of the articles are possessed by the school.

About 40 per cent of the schools replying reported mathematical reference libraries varying in size from two or three volumes to several hundred volumes. The median size is 50 volumes

Three-fourths of the schools have geometrical models and drawing tools for use with the blackboard. A measuring tape is part of the general equipment of half of the girls' schools and two-thirds of the others. About one-third of the girls' schools and more than half of the other schools include a plumb line in their lists of mathematical apparatus. One-third of the schools for boys and for both sexes and a small proportion of the girls' schools have engineers' transits and nearly as many have sextants. A few schools report other surveying instruments which were not included in the printed list. About one-fourth of the schools have slide rules, coordinate blackboards, and spherical blackboards; a smaller proportion have portraits of mathematicians and a very few have lantern slides illustrating mathematical subjects. On the whole the boys' schools have more extensive general equipment than the others.

In practically all of the schools each pupil is supplied with a ruler and compasses. In a majority of the schools he has also triangles, protractor, dividers, and squared paper; and in many schools a ruling pen is added to the pupil's individual equipment.

With the exception of the possession of a mathematical library, which was indicated by recording the number of volumes, an item of equipment not crossed out stands as a possession of the school. This fact doubtless makes the foregoing proportions too high.

Time of emphasis upon logical relations.—In accordance with the suggestion of the central committee in its report outlining the plan of the investigation, a question was included in the principal questionnaire asking at what stage (grade) emphasis is shifted from manipulative skill to the understanding of logical relations. Only about 40 per cent of the returns gave answers to this question, the replies from the girls' schools being especially meager. The question seems to be too vague to make the reports of much value.

Every grade from the first year in the primary school to the beginning of the college course is mentioned as the time for emphasizing logical relations. Many lay stress on reasoning from the beginning of the course, and others say that the change of emphasis comes about "gradually," "at no definite time," or "varies with the class."

The grade mentioned most often as the point of transition in the reports of each class of schools (boys', girls', and coeducational) is the second year of the four-year high-school course, when pupils are about 15 years of age. The general tendency seems to be to make the change of emphasis earliest in the girls' schools and latest in the boys' schools.

EXAMINATIONS AND TESTS.[1]

Examinations given by the school.—Formal written examinations are given by nearly all the schools, varying in frequency from twice a month to once a year. There are two well-marked tendencies, one toward short monthly examinations and the other and more general one toward longer examinations two or three times a year. Two examinations are commonly given in the girls' schools and three in the boys' schools and coeducational schools.

The examinations vary in length from 30 minutes to 240 minutes. In the girls' schools and coeducational schools monthly examinations commonly last about 45 minutes, and term or semiannual examinations 2 hours. In boys' schools there is greater variation in practice, but the tendency is toward longer examinations, 3 hours being nearly as common as 2 hours for term examinations.

In a few schools promotion is dependent solely upon the passing of these examinations, but the common practice in calculating the pupils' standing is to give examinations one-half or one-third of the total weight. The papers are in almost all cases set and marked by the teacher.

In addition to the formal examinations written tests are given in most schools, sometimes weekly, but more commonly once or twice a month. These tests occupy in most cases about 45 minutes.

Purpose of examinations given by the school.—The purposes most generally recognized for examinations given as a part of the school

[1] Data of supplementary questionnaire.

work are to serve as a spur to pupils and as an incentive to review and organization of subject matter. The latter purpose is considered by many to be the most important. Most of the teachers consider examinations of some value as tests of a pupil's ability and progress, but in both these respects many point out that such tests are often misleading. There is nothing in the replies to indicate that examinations are used to measure a pupil's progress with any degree of exactness. A majority of the teachers consider that examinations act as a spur to teachers and serve as a test, for their own benefit, of the efficiency of their work. Many, however, do not think highly of this purpose of examinations. About half the replies approve of examinations for the purpose of comparing the relative standing of different classes. The purpose securing the least support is that of testing the efficiency of the teacher for the enlightenment of school officers.

Ill effects of examinations given by the school.—In answer to the question, "What ill effects have you noticed of the examinations given in the school by teachers or school officers?" Forty-one schools replied "none" and 30 gave no answer, which in most instances is probably equivalent to the answer "none." The ill effects most frequently mentioned are nervous excitement (28 cases), "cramming" (15 cases), tendency to place a wrong interpretation upon the value of examinations, including working for marks (14 cases), discouragement of weak students (10 cases), and temptation to dishonesty (9 cases). Most of the reports referring to nervous strain came from girls' schools; only two were from boys' schools. In many instances the teachers reporting emphasize the fact that the ill effects are confined to a few individuals.

Modifications of school examinations are suggested in only a few of the replies. These include exemption from examinations for pupils of high standing, increase in number and decrease in length of examinations.

Admission of students to higher institutions.—Two methods are in common use for determining a student's fitness to enter college, namely, (1) examinations by the college or by the College Entrance Examination Board and (2) a certificate furnished by the school of the completion by the student of the requirements prescribed by the college. Among the 113 schools reporting on this matter, practice seems to be about equally divided between the two methods. In 22 schools all candidates take examinations, in 17 all enter on certificate, while in the others the two methods are combined in all proportions. In New England the examination method is the common one, and in the North Central States the certificate. In the Middle Atlantic States a majority of the schools send more than half their students to college "by examination," but many use the certificate freely.

Among the girls' schools there appear to be two distinct groups. In the larger group the certificate is used freely, in the smaller one it is never used.

The variation in use of the different methods is due in part to the demands of the colleges, since a few of the largest institutions admit students only by examination. It is evident, however, that many schools are not in favor of the certificate plan, and prefer to have their students take examinations.

Effect of college entrance examinations.—A large majority of the schools report that the course of study has been affected by college entrance examinations. The most general criticism is that the courses are overcrowded.

Only a few mention any effect on methods of teaching, but nearly all of these agree that much time is devoted to drill in preparation for the examinations which would otherwise be omitted. The papers of previous years are used for practice. One school required its students to spend alternate Saturday mornings after New Year in taking these practice examinations. A few say that the restrictions imposed upon the freedom of the teacher by the necessity of preparing for the examinations tends to make his work mechanical.

Most of those who refer to standards of work think that they are raised by the examinations. A few think that they are lowered. Others say that they furnish a definite standard.

Effect of the certificate plan.—A majority of the schools which send pupils to college "on certificate" report that no changes have been made in course of study, methods of teaching, or standards of work as a result of inspection of the school or scrutiny of the records of its graduates by college officers. Several, however, say that the work has been made more thorough in response to these checks. Some schools keep a record of the college work of their graduates who have been entered "on certificate," and several make a practice of certifying only pupils who attain a high standard.

One teacher says that the success in college work of the certified graduates of his school encouraged him to depart further from conventional methods and courses. Another says that the responsibility of a school for the college work of a graduate whom it has certified stimulates teachers to strive to give their pupils a real grasp of the subject rather than a temporary knowledge such as serves for passing an examination.

MATHEMATICS AND COEDUCATION.

As the practice in the private secondary schools of the United States is about equally divided between coeducation and separate education of the sexes, this field seems to offer a peculiarly favorable opportunity for studying the relation of coeducation to mathematical

education. The practice in the different types of schools has, therefore, been presented in the foregoing discussion in relation to most of the points considered.

To supplement the statistics bearing upon actual practice, it was felt that the opinions of the teachers as to the bearing of coeducation upon the effectiveness of mathematics teaching would be of value. Accordingly, the supplementary questionnaire for coeducational schools contained questions calling for such opinions. Thirty-six replies were received.

Nine of the teachers answering the questions had taught in coeducational schools for not more than 5 years; 13 from 6 to 10 years; and 13 over 10 years. Only 6 had taught separate classes of boys, and only 3 separate classes of girls, all of whom were included among the 6 who had taught boys alone. The fact that so few of the replies came from teachers who have had experience in teaching the sexes separately evidently diminishes the value of the testimony.

In all the schools represented by the replies coeducation had been in use from the establishment of the school; but one reported having separated the sexes for class instruction during recent years. Two schools besides the one just mentioned reported that there is some prospect of separation of the sexes in class work, in the interests of a better adaptation of rate of work, character of subject matter, and methods, to peculiar characteristics of the sexes. None of the other teachers see any prospect of the modification of the present plan.

Teachers were asked to compare the sexes with regard to their success in passing the requirements in the various mathematical subjects. In arithmetic 20 report no difference, 7 state that boys are more successful, 4 that girls are more successful; in algebra 21 find no difference, 6 consider boys more successful, 5 consider girls more successful; in geometry 13 report that the sexes have equal success, and 19 consider that boys excel; in trigonometry, 9 report no difference, and 14 consider that boys are more successful. Several said that, while boys are as a rule superior as mathematical students to girls, there are frequently brilliant students in mathematics among the girls.

In regard to the relative tendency of boys and girls to choose elective courses in mathematics, the answers indicate that boys are much more inclined to elect such courses than girls are.

In accuracy of intuition in regard to mathematical relations, 6 of the teachers report no difference in the sexes, 1 considers girls slightly superior, and 19 boys superior.

In skill in formal processes, 11 find no difference, 8 find boys superior, 10, girls superior.

In grasp of logical sequence, 6 report no difference, and 25 boys superior.

In ability to solve "original theorems," 28 say that boys are superior, and 3 report no difference.

In answer to the question, "Do you think that the subject matter of high-school courses in mathematics should be the same for both sexes?" 14 replied "Yes," 9 replied "No," and 7 replied "Yes," with qualifications. The differences in subject matter for boys and girls regarded as desirable by some teachers are: (1) Requirements in algebra and geometry should be less for girls than for boys. (2) Solid geometry and trigonometry should be omitted or made optional for girls. (3) The applications should be such as appeal to the interest of both sexes.

Only one difference in methods of teaching the sexes was suggested and that was mentioned by only one teacher; namely, that mathematics be developed empirically rather than logically in the instruction of girls. It is interesting to note in this connection that one teacher, arguing from the same characteristics as shown by the sexes, would give the girls special training in logical reasoning.

To sum up, most of the teachers who gave testimony agree that there is a difference in the mental traits of boys and girls and in their ability to do different kinds of mathematical work; but a decided majority do not consider that these differences are sufficiently great to warrant separate class instruction. About a fourth of these teachers would provide for sex differences by a slightly different treatment of pupils in the same class and by making part of the work elective; another fourth feel that the differences are too great to permit the most effective work unless separate classes are provided.

AIM OF INSTRUCTION IN MATHEMATICS.

With a view to gaining some insight into the attitude of teachers of mathematics toward their work and to learning the extent to which they attempt to analyze their problem, the supplementary questionnaire included a list of questions on the aims of instruction in mathematics and the means employed to attain these aims. The list included questions on the general aim of mathematics teaching, the aims especially served by the different mathematical subjects, the inclusion of individual topics in the curriculum for specific purposes, the relation of course of study in mathematics to vocational training, and the value of mathematics for mental discipline.

Aim of instruction in mathematics in general.—In presenting the question on the general aim, nine statements were given of which teachers were asked to cross out those not recognized, a method which gives no means of discriminating between controlling aims and those having only a slight influence. This defect is remedied to some extent by the questions on the relation of particular subjects and topics to the various aims.

Of the 136 replies, all recognize *mental discipline* as one of the aims of mathematics teaching, and all but three work for the *development of an accurate conception of space and form.* Nearly all approve of *preparation for more advanced work in mathematics, preparation for studying other subjects, the teaching of mathematical truths for their own sake,* and *the cultivation of an appreciation of the importance of mathematical knowledge in modern life.* Only eleven schools are able to ignore *preparation* either *for examination* or *for requirements imposed by outside authorities.*

The *preparation for vocations* is the aim least generally recognized. Ninety per cent of the boys' schools recognize this aim, but only a little more than half of the girls' schools regard it.

Predominant aims in teaching special subjects.—About one-fifth of those reporting apparently find it difficult to select the aims chiefly served by the different mathematical subjects, for they pass the question by. The replies represent a very wide variation of opinion, every one of the aims being mentioned by several as important in connection with each of the subjects. Very few consider any one aim as sufficiently influential in teaching any subject to warrant its standing alone as the predominant aim.

A rather marked difference exists in point of view of the teachers in girls' schools on the one hand and those in boys' schools and coeducational schools on the other. Among the girls' schools, mental discipline is most generally regarded as a predominant aim. In the other schools, vocational training and preparation for advanced work in mathematics are mentioned most frequently in connection with arithmetic and trigonometry, and preparation for study of advanced mathematics and other subjects in connection with algebra. In geometry, mental discipline takes first place, and the development of accurate conceptions of space and form stands second in the reports from all three types of schools.

Topics included in the course to serve specific aims.—About 30 per cent of the reports make no attempt to answer the question on the relation of aims to particular topics of the course. The others represent the greatest variety of opinion. Many teachers apparently have no specific purpose in mind in teaching any topic and there is little agreement as to the topics which best serve a given purpose. Every one of the nine aims listed in the paragraph on aims in general is given by several teachers as the chief purpose for including *graphs* in the course. One can hardly avoid the conviction that custom as expressed in college-entrance requirements and in textbooks is the chief factor in determining what topics shall be taught.

Proposed requirements in mathematics for various classes of students.—In the opinion of the majority of teachers there is need of very little differentiation of mathematical courses in accordance with vocational

intentions or plans for higher education. Arithmetic, algebra, and plane geometry would be required for all classes of students, except the boys preparing for technical schools, who, in the opinion of the majority, have had enough arithmetic in the elementary schools. They should, however, have solid geometry and trigonometry in addition to the other subjects. While this represents the consensus of opinion several modifications are strongly supported. For college preparatory students of both sexes and for boys preparing for law and medical schools nearly half the teachers would omit arithmetic as a requirement in the secondary school. More than a fourth of the replies call for solid geometry as a requirement for boys preparing for college, and advanced algebra for boys intending to enter technical schools. For boys anticipating a business career and for girls whether preparing for technical schools, for commercial positions, or for domestic responsibilities a strong minority would require neither algebra nor geometry. In the case of the girls this view is supported by from 20 per cent to 40 per cent of the replies. A few would make the geometry concrete for girls not preparing for college and many would treat the subjects differently for commercial students (especially girls). The modifications suggested relate chiefly to shortening the courses, omission of theoretical discussions, and emphasis upon practical applications.

Analytic geometry and elementary treatment of the calculus are proposed by only four or five teachers as part of a course for boys preparing for technical schools.

Following is a summary of the replies in tabular form:

Proposed requirements in mathematics for various classes of pupils.

Proposed requirements.	Number of teachers recommending certain studies for—							
	Boys for—				Girls for—			
	College.	Technical schools.	Business	Professional schools.	College.	Technical schools.	Commercial schools.	Home duties.
Arithmetic	40	30	63	42	52	69	82	77
Elementary algebra	80	80	69	73	98	64	65	74
Advanced algebra	6	24	2	3	5	3	1	1
Concrete geometry						9	5	7
Plane geometry	80	80	61	71	98	59	53	65
Solid geometry	21	67	9	15	10	3	2	2
Plane trigonometry	11	60	6	12	8	2		1
Analytical geometry		4						
Elements of calculus		5						
Total number of replies	80	80	77	75	98	85	88	92

Mental discipline.—It has already been said that all the teachers reporting recognize mental discipline as one of the aims of mathematics teaching. As psychologists are claiming that too much

reliance has been placed upon the supposed value of study of a single subject for general improvement of mental processes, the opinions of teachers of mathematics upon this point seem pertinent to a discussion of aims.

To the request "If you regard mental discipline as an important aim, explain as fully as possible what you mean by mental discipline," nearly nine-tenths of the teachers give some reply, but very few attempt to analyze their conceptions. Mental discipline is, in most cases, described in vague, general terms, loosely applied, representing all sorts of mental and even moral qualities which are believed to result from a discipline of the mind. Nearly all have a firm conviction that general abilities are gained through exercise of the mind upon a particular subject, especially mathematics.

A composite of the replies shows that mental discipline is considered to be that which produces an improvement in intuition, judgment, memory, imagination, intelligence, reason, mental powers, reasoning powers; or an improvement in ability or power of mental concentration, initiative, sustained effort, analysis, generalization; or an improvement in ability to think rapidly, clearly, independently, logically; to recognize the essential elements in a problem, to note resemblances and relationships, to grasp and apply principles, to understand cause and effect. One of the most generally approved results of mental discipline is the ability to express thoughts clearly, concisely, and accurately. In a few cases mental discipline is described as the formation of habits; habits of mental concentration, of industry, of accuracy in thought and expression.

Following are a few examples of more definite analyses of the authors' conceptions of mental discipline:

Mental discipline is that process of mind which (1) recognizes there is a problem. (2) Wills that the problem be solved. (3) Perseveres until the desired goal is obtained.

Results of mental discipline are—

ability to observe well, to make correct records, written or in memory, of things observed, to sift data or evidence, to draw correct inferences, to state these inferences in clear language.

In only a few cases, such as the two following, is the influence of modern views of formal discipline apparent:

The modern psychologists have made this a difficult thing to do (explain meaning of mental discipline), but the habits of making exact statements, of finishing a piece of work in hand, of seeking the proof for each statement, not only ought to be valuable but I am convinced are valuable.

Mental discipline through mathematics for mathematical work consists in the acquirement of mathematical facts and ideas, processes, methods of solution, and standards of accomplishment. Mental discipline through mathematics for other kinds of work consists, I think, in development or strengthening of certain ideals or standards,

such as logical perfection, mastery of difficulties, etc. General power is not necessarily gained by the study of mathematics, but the student may be so impressed by the logical perfection of mathematical reasoning as to consciously test his thinking on other subjects by similar standards, and the sense of mastery experienced in mathematical victories may give confidence in attacking other difficulties.

About half of the teachers consider mathematics superior for mental discipline to all other subjects. A considerable number consider it superior for certain kinds of discipline, usually for improvement in logical reasoning or in accuracy. A few qualify their approval, saying that mathematics is superior to some subjects or for some minds. About 10 per cent of the teachers do not regard it as superior to other subjects and one says in answer to the question, "Decidedly no."

The nature of the superiority is usually stated in terms of the desirable qualities of mind which are said to be produced more readily through mathematics than through other subjects. One says it "requires thought." The superior quality of mathematics most frequently referred to are its definiteness and the absolute trustworthiness of its principles. The simplicity of its data and the ease of checking results make this the best medium for training the student in logical reasoning. On the other hand, it is said to call for greater mental effort, and therefore to result in greater mental power.

DISTINCTIVE FEATURES IN A FEW INDIVIDUAL SCHOOLS.

The following statements are added to the report, as originally prepared, in response to the feeling that the omission of all reference to experiments and attempts to improve prevailing practice would be misleading as a statement of the situation in the private secondary schools and also neglectful of the opportunity of acquainting teachers with such movements. The omission of this phase of the subject in the original draft was due to the impossibility of making a comprehensive statement in regard to it, and in employing this makeshift the committee is anxious that its relation to the rest of the report be clearly understood. We have no means of knowing how generally the ideas expressed in these six statements are affecting courses in the schools of the country, but it is certain that the two ideas which are here so prominent—unification of the various elements of the course and application of mathematical principles in daily life—are receiving much attention. We do not mean to imply that these are the most important movements now going on, and we are particularly desirous that it be understood that these schools have not been singled out as the best representatives of progressive teaching. In the absence of material for an adequate statement of movements for improvement, these statements are offered merely in proof of the fact that such movements are in existence and in the hope that the plans described may be of service to other teachers.

PRINCIPLES UNDERLYING THE COURSE IN MATHEMATICS AS STUDIED IN THE DETROIT HOME AND DAY SCHOOL, DETROIT, MICH.

[This part of the report was prepared by S. A. Courtis, of the Detroit Home and Day School.]

The Detroit Home and Day School is a private school for girls. The activities of the school extend from the kindergarten through college preparatory work, and as the period of secondary education has been lengthened to 5 years, the courses provide for 15 years of continuous school life. About half of the 362 pupils enrolled during the present year (1911) are in the five grades of the academic department. The number of teachers and officers is 42. While nominally a commercial enterprise, practically the school is the life work of a single group of persons, the expression of a consistent set of educational ideals.

The school is educationally independent. While in all its college preparatory work its courses and standards are determined almost wholly by college-entrance requirements, it has, nevertheless, a sufficient body of pupils not going to college to warrant the close study of its own problems and the modification of its courses and methods to meet the needs and demands of the community. The school must be regarded, therefore, as an educational unity, complete in itself.

The organization of the school is along departmental lines, the departments being English, history, language, science and mathematics, and physical training. Heads of departments are free to work out the general aims of the school in ways best suited to the subject and conditions of the department. Unity is secured through unity of aim rather than through rigid adherence to a fixed system.

The vital characteristics of the school are three: Continuity, coordination, and correlation; continuity, because the courses in each department, from lowest to highest, form an unbroken series under one control; coordination, because in the same way all departmental work and all the life of the school from the lowest to the highest is under one control; and correlation, because from the viewpoint of those in control education is regarded as a whole, not as many separate wholes.

Taking up now the consideration of the mathematics course, the first point to be noted is the departmental unity made possible by the general organization of the school. So completely are the courses in mathematics the expression of a single aim that it will be impossible to consider the high-school work apart from the general plan. There is no break or change in passing from the earlier to later grades, from arithmetic to algebra and geometry. So completely are the subjects merged that the work of the high-school years is better described as a three-year course in mathematics than as specifically either algebra or geometry for any one year. Further, the direct preparation for the high-school subjects, geometry, for instance,

runs back to the earliest work. An increasing amount of geometric material is used in every grade, and the study of formal, demonstrative geometry is but the logical outgrowth of the preceding work. In the same way graphic work in algebra, or the use of the literal notation, is but the final culmination of an extended series of exercises through many previous grades. On the other hand, direct provision is made for review and drill upon certain topics of arithmetic in the high-school work. Further, in selection of subject matter, choice of methods, shift of emphasis, and remedying of defects, the point of view is that of the course as a whole rather than that of any single grade or division of subject matter.

A second point to be noted, also a direct result of the general principles of the school, is the correlation of the work in mathematics with the work in other departments. The merging of the departments of science and mathematics into one has resulted in the introduction of much laboratory work and science material into the work at all points, particularly into the intermediate grades in arithmetic. The laboratories of the school are in use nearly as much by the classes in arithmetic and algebra as by those in science. Material for problems is also drawn from other school subjects, geography, history, etc., from the activities of the school, and from daily life. A graded course in actual business and banking is a feature of the intermediate arithmetic work. In every way the effort is made to have the child realize the meaning of the work and its bearing on his daily life.

Pedagogically, the work of the department is based upon the condition that there are three steps in the natural growth of intellectual powers: The acquisition of basic, sensory experience; the development of control through use; and, finally, the formulation of the abstract principles involved. The attempt is made to have the children meet each new topic first in a concrete setting, as a problem for which no method of solution is known. The class work and laboratory then develop the method of attack and solution, and finally the knowledge and skill attained are tested in concrete problems out of which new topics arise.

In addition to conforming to the natural steps of intellectual growth, there is also the problem of adjustment of the character of the work to the development of the growing child. The courses of the school provide adequate training at every stage of growth. Not only is each topic met in at least three successive grades (the first year, incidentally; the second year, as the important topic of the year; the third year, for analysis and formulation, for learning of rules and for drill in the abstract principles involved), but the course covers the same ground three times, from different viewpoints, and passes by easy, connecting steps from the observational, concrete work of the kindergarten to the formal and abstract demonstrations of algebra and geometry.

From the foregoing it will be seen that the emphasis is placed upon the reasoning rather than upon the mechanical side of mathematics. Mention has already been made of the use of laboratory exercises, but this phase of the work needs special emphasis, for out of it all mathematical work proceeds and to it all knowledge and skill returns. Every effort is made to insure the intelligent use of mathematics as a tool. Opportunities are thus provided for the child to weigh values and learn through his own mistakes. For it is believed that understanding knowledge comes by use and not by instruction; that the child must have a chance to learn through his own struggles toward success.

On the other hand it is recognized that the growth of the child in accuracy and knowledge is of equal importance with his growth in understanding. Constant review of the old in the new is provided and there is systematic effort to let no part of the old be lost through disuse. Adequate and rigid drill, but drill in its proper place, after mastery through use, insure a high final efficiency. The extensive laboratory work and the provision for drill are both distinctive features of the course.

One further topic will be discussed, the aim of the mathematics courses, particularly in high-school years. The work of the department is organized around the belief that courses in algebra and geometry have a place and a function that can be filled by no other subject, i. e., to make plain the essential nature of the thinking processes itself. The value of mathematics for this purpose is unique because it alone can provide mental environment completely under control and completely devoid of emotional content. In such an environment only can an individual clearly see the relations between the various mental steps or acts which make up thinking. There is no question of transfer here. The child that learns to think in mathematics does not necessarily learn to think in other subjects, but the child that learns how he thinks in mathematics and that is taught to think in other subjects soon learns that all thinking is conducted on the same general plan. The courses in mathematics are planned to bring to consciousness the general features of all inductive and deductive thinking that out of any and all the mental activities of the individual there may issue the sooner that general method of behavior in the presence of a real problem that is characterized as "general ability." To this end the exercises of the school lay more emphasis than usual upon initiative and executive ability, upon original discovery and analytical thinking, upon recognition and formulation of relations as well as upon the deductive application of proved or accepted principles. As a result the mathematics courses have a vitality and a functional relationship to the rest of school work which was not possible under the older system.

Other features of the school are worthy of discussion, particularly the attempt of the school to study itself and to standardize its work through comparative tests given year after year under uniform conditions. The features mentioned, however—unity, correlation, and the principles back of the courses and methods—are those which distinguish the school from others of its kind and which have produced whatever of merit there may be in the activities of the mathematics department.

UNIFICATION OF ELEMENTARY MATHEMATICS.

[This part of the report was prepared by John S. French, principal of the Morris Heights School, Providence, R. I., and relates to certain experimental work in that school.]

The Morris Heights School, a boarding school for boys, is, primarily, a preparatory school for institutions of higher learning, its course extending over a period of 12 years, beginning with the first year of formal school work and concluding with the preparation for collegiate or technical training; pupils may thus retain membership in the school for a long period of time and benefit from the advantages of a continuous and progressive course of study. Hence, with particular attention paid to the mental condition of the pupil as a result of growth and with the opportunity of eliminating irrelevant material the work is so designed as to lead ultimately to preparation for higher instruction.

The general idea underlying the teaching of mathematics is the development of the scientific aspect. This does not lose sight of the fact that the mathematics of the elementary and secondary schools is limited to the technique of the subject, nor does it pretend to include the notion of a purely scientific development which involves a mastery of hypotheses far from the reach of younger minds. It does include in its scope the development of a working knowledge of the fundamental concepts where an appreciation of the relation between the different branches is emphasized for the purpose of unifying the work.

The time for the introduction of any topic into the curriculum is based on the principle that any topic entering into the sphere of a pupil's intellect should be taught him at once, in order to fix in his mind correct ideas about it.

To this end science is begun in this school in the fifth year (corresponding to the fifth grade), in which the pupils are taken into the laboratory and are shown by experiment and observation a large and, to be sure, unrelated group of facts in the realm of natural phenomena. In connection with this study there is gradually worked in, as a tool for getting results, the elementary concepts of form and their relation to number. Thus is laid gradually and un-

knowingly to the pupil the foundations of science, tending to a unification of subject matter later on.

To illustrate this work it is necessary to cite a single example only —the different methods of heating and ventilating buildings are studied, the pupils inspecting them under supervision. As an instance, they are required to draw a diagram of a heating system, to show the indirect method of heating by steam, and particularly to show its efficiency in ventilation; in addition to this certain dimensions are given, and from these and from statistics on coal consumption they are required to figure roughly the amount of coal necessary for heating the building. This work is continued in the sixth, seventh, and eighth years where the geometry is wholly inventional and the symbolic forms of algebra are used as a means for solving problems. The pupils continue the work in graphics by plotting barometric and thermometric readings, thus getting the first real taste of a continuous variation.

Algebra and geometry are taught simultaneously in the third form (first high-school year). The transition from inductive to deductive geometry is in no way abrupt; but the notion of one dimensional and two dimensional figures is carefully explained and with it the notion of equations of one and two unknown quantities, thus making the analogous relation between the two evident. The practical aspect of the two subjects is emphasized and their correlation with physics is continued.

In the study of formal deductive geometry the tendency is to introduce the principle of continuity as often as possible and thus avoid the difficulties to be encountered later on, resulting from the archaic methods of treating isolated topics; it also tends to make the work conform to the general results obtained in algebraic processes. This involves, among others, a consideration of magnitudes and directions in the study of metrical properties and also the geometric interpretation of real and imaginary, equal and unequal numbers, and is shown in the study of propositions involving secants and tangents, parallels, perpendiculars, similar figures, etc.

In the treatment of originals particular care is paid to the gaining of correct notions of analytic and synthetic methods of proof, and the equivalency of equations in transformations is very carefully treated.

In the more advanced courses the fundamental notions of the representation of a condition algebraically by means of the equation and geometrically by means of the locus, and the expression of one in terms of the other, are taken up where the representation of the straight line and circle in terms of algebra by its required number of

parameters is touched upon. Thus is brought in, in their true perspective, the ideas of functional dependence and functional variation.

The ideas of maxima and minima are studied in the graphical representation of functions, and the algebraic definition of a derived function is here made use of. In this work the pupil is taught how to express the conditions of his problem in algebraic forms and from this to plot the curve of the function whence the maxima and the minima points must correspond to the values obtained from equating the first derived function to zero and solving.

The trigonometric functions are brought into the geometry and their use in the generalized Pythagorean theorem is a tendency toward generalization; also, the idea of the vector quantity is frequently touched upon, so that when the pupil comes to the study of velocities and forces the expression, graphically, of resultants in terms of components presents no grave difficulties.

The notion of a limit is treated, not with strict mathematical rigor, but with the idea of creating a working knowledge of it gained inductively through concrete examples; it makes clear the similarity of incommensurables and irrationals by means of a graph where segments of the line have their distances measured in such a way as to give a clear representation of the limits in both cases.

In the constant use of symbolic forms such symbols are chosen as are applicable to the work at hand and in accord with usages higher up.

These somewhat disconnected statements concerning the work done in elementary mathematics are offered to show that the general tendency of the work is toward unification, having as an ultimate end the easy assimilation of the more difficult and severely rigorous methods employed in the teaching higher up. It is my opinion, based on experience, that the teaching of mathematics as an organic unity in which the various branches stand in the closest relation of interdependence is the only rational method of presentation, and, further, I am convinced that irrespective of the future work of the pupil he can be carried on with equal efficiency by this method of breaking down barriers and treating the different branches simply as multiform ways of expressing concepts which are closely allied as different modes of combining these concepts and as different representations by means of distinctly differing symbols, and these because the same laws of exact reasoning fundamentally govern all the branches.

This scheme of presentation is of value not only in adding to the efficiency of the pupil, but in making possible the covering of a wider area, even to including in his preparatory course a major portion of the work of the freshman year in college and this with an aggregate appropriation of 20 periods for the four-year course of the preparatory school.

COURSE OF STUDY—"MIXED MATHEMATICS."

[This part of the report was prepared by Prof George W. Myers, of the University of Chicago, and relates to certain experimental work in the University High School, Chicago, Ill.]

The School of Education of the University of Chicago consists of four distinct parts, the graduate school of education, the college of education, the university high school, and the university elementary school. In this complex of departments the university high school, a coeducational institution of 600 pupils, undertakes to play a threefold rôle. It seeks to carry forward the best type of general secondary education possible to its favorable situation between a carefully planned and executed elementary school and a strong and sympathetic university, supervised by a collegiate faculty of education and stimulated and aided academically in each of its departments of study by a strong cognate university department. It functions as a laboratory for students of the department of education for both observation and practice teaching in preparation for actual teaching in secondary schools and academies. It also undertakes to furnish a high order of preparatory training for colleges and universities. It has also strong departments of manual training and of applied arts.

Pursuant to this threefold office the department of mathematics of the school of education a few years ago undertook the double duty of endeavoring to work out in the classes into teachable form a body of mathematical subject matter of high educational quality that should jeopardize neither the interests of mathematical training nor of university entrance and at the same time to make the classrooms function as exhibit and laboratory rooms for teachers in training. After considerable preliminary experimenting it was determined to undertake to organize a body of mixed mathematics around a central line of algebraic notions for first-year students and to make geometrical ideas the controlling theme for second-year students. As to whether the plan of unified or mixed mathematics is to be carried through the third and fourth years or the plan of topical treatment is to be reverted to was left to be determined on the basis of experience with the classes of the first and second years. The first two years of the course have been worked out and the material is published in the texts entitled *First-Year Mathematics* and *Second-Year Mathematics* by the University (of Chicago) Press. Experience is now being sifted, analyzed, and studied, and it is believed the last two years of the course will soon be completed, though the precise nature of this part of the course can not yet be stated. With the part of the course already completed experience is highly encouraging.

Each teacher of the high school has participated both in the authorship and in the trying out of the material. On the whole the most encouraging feature of the experiment is the improvement that it has wrought in the teaching. The spirit of unity it has begotten

in the corps of instruction through cooperation on a common problem can be fully appreciated only by schoolmen who have seen the waste due to lack of cooperation.

With the cooperation of the high-school principal it has been possible to give each teacher at least one first-year class, one second-year class, and one or more third or fourth year classes. This vertical division of the teacher's duties brings him into close touch with all parts of the experiment and with every grade of maturity in the high school. This enhances interest by reducing the mere routine of both teaching and learning.

A fourth-year review course, consisting of one hour per week, is now required of all high-school students who are intending to obtain a diploma with certification to college. This course reexamines and renews the hold on the work covered during the first two years and gives to this work something of a scientific classification and treatment. The purely formal aspects of the work of algebra and geometry are given strong emphasis in this review. The work of the first two years together with this fourth-year review course constitute the required work of the school.

In the third year parallel courses are given in plane trigonometry and third-year algebra throughout the year, two hours per week being given to one subject and three to the other during the first semester, the time ratio being reversed the second semester. At present solid geometry and college algebra are given as separate subjects in the fourth year to those who elect them.

The method of procedure of teaching combines the laboratory, heuristic, expository, and recitation methods. New developments are worked out with the classes, sometimes heuristically, sometimes by the laboratory plan, and now and then by the recitation, quiz, and lecture plan. Home assignments are such as to call for further developments of fairly fully developed theory or examples to illustrate and emphasize theory. The first two years do not aim so much at thoroughness as at gaining a first hold on the coarser outlines of algebra and geometry. The later work places more emphasis on thoroughness and accuracy, though it does not, as is too often the case, lay great stress on mere "labial" precision.

The topics in order that are covered the first year are:

I. General uses of the equation based on the notion of balance of values.
II. Uses of the equation with perimeters and areas; related geometry.
III. The equation applied to angles; related geometry and arithmetic.
IV. Positive and negative numbers; related arithmetic and algebra.
V. Beam problems in one and two unknowns; related mechanics.
VI. Problems on proportion and similarity; related geometry.
VII. Problems on parallel lines; geometric constructions.

VIII. Fundamental operations applied to integral algebraic expressions.
IX. Practice in algebraic language; general arithmetic.
X. The simple equation in one unknown.
XI. Linear equations in two or more unknowns; graphic solutions.
XII. Fractions.
XIII. Factoring; quadratics; radicals.
XIV. Polygons; congruent triangles; radicals.

In the second year the topics covered are:

I. Congruency of rectilinear figures and circles, with related algebra.
II. Ratio, proportion, and similar triangles; related algebra and arithmetic.
III. Measurement of angles by arcs of circles; related algebra; use and reduction of radicals.
IV. Similarity and proportionality in circles; quadratic equations; solution by formula.
V. Inequalities in triangles and circles; algebraic inequalities, indeterminate equations; discussion of roots; simultaneous quadratics.
VI. Areas of polygons; use of algebraic formulas and expressions.
VII. Regular polygons in and about a circle; use of formulas and equations.
VIII. Problems and exercises in graphic and geometric algebra; radical equations.

We have as yet no statistical measures of the precise increase of output of this *modus operandi* with high-school students, but there are some improvements that are none the less real because they have not been reduced to figures. A few of them, confessedly of a qualitative nature, I think are worthy of mention in this report:

1. An increase in mathematical interest, earnestness, and spirit among early pupils.

2. A genuine belief attained earlier than formerly among pupils as to the real worth of mathematical study.

3. An improvement in independence and solidity of mathematical thinking among pupils of the first and second years.

4. There is less of the disposition manifested now than formerly by pupils to learn a mathematical study mainly to pass an examination.

5. Pupils try more persistently to check and guarantee algebraic results by some sort of geometrical means than was formerly the case.

6. Pupils ask more frequently than formerly in friendly interest what advanced subjects are like, and manifest a desire to take more rather than less mathematics.

7. Pupils acquire more of the all-round benefits of mathematical education. They are better balanced mathematically than formerly.

Most teachers will agree that such results as these indicate a distinctly tonic influence upon the mathematical thinking of boys and girls.

THE PLANE GEOMETRY COURSE IN THE POLYTECHNIC PREPARATORY SCHOOL, OF BROOKLYN, N. Y.

[This part of the report was prepared by Eugene R. Smith, of the Polytechnic Preparatory School, of Brooklyn, N. Y.]

The Polytechnic Preparatory School is a day school having a secondary department of 350 boys. As the pupils enter from various private and public schools, the problem of the school is not simplified by uniform preparation. Its classes are small, the average size of the mathematics classes for the year 1909–10 being 18. The departments are highly centralized, the head of each department having both the authority and the responsibility for his department, and being subject only to the headmaster's general supervision of the school. The department of mathematics is composed of five men, none of whom teaches any other subject.

Elementary and intermediate algebra and plane geometry are required subjects, and are allowed five 43-minute periods per week for the first three years of the course. The first year is given up to elementary algebra, but the other two years have a combination course of algebra and geometry, about three-fifths of the time being given to geometry. The object of this simultaneous arrangement of courses is twofold: Increased interrelation of the subjects, and economy of time through continuous treatment of each subject from the time of its introduction until its completion.

In geometry, the use of a textbook containing the proofs of the propositions has been abandoned, and the classes are supplied with a text containing definitions, statements of propositions, discussions of method, including summaries and other helps to correct attack, and exercises.

It is believed that the chief aim in the teaching of geometry is the development of the power of logical thinking, and that this power, like any other, improves with practice. The intention of the course is, then, to discourage rote memorization of proofs and to encourage independent thinking. The propositions are all treated as "originals," or subjects for investigation, and sometimes statements of relationships are deferred until the facts needed to establish them have been discovered. Class development of propositions and exercises is by the "question and answer" method, the teacher asking questions suited to the ability of different pupils, and aimed to lead to a correct method of attack. Any ordinarily intelligent pupil who knows the preceding part of geometry as this requires him to know it, can do a good share of his own thinking, and while he must have guidance from the teacher, he does not need to be told things outright.

Besides class development of new propositions, there is much oral discussion and cross-examination on the relations of theorems,

their uses, methods of attack for various kinds of exercises, and other related topics. Propositions and exercises are assigned to be worked out at home or in study hour, and a good deal of written work is assigned; this written work comprises careful drafts of propositions and proofs, more or less condensed, of many exercises. It will be seen that this is a combination of the heuristic method and the genetic mode; it has, however, become customary to speak of it as the "syllabus method."

I have said that the pupils must know the preceding part of geometry. In no other way, I believe, does a pupil come to know so thoroughly just what he has had and for what he can use his knowledge as the subject is developed, the theorems are grouped, the use of each and its relation to those that precede and those that follow it are discussed, and an attempt is made to have the propositions classified in the mind of each pupil according to their uses. With this equipment a pupil who wishes to prove a proportion, to demonstrate a length relation, or to discover any geometrical truth, is able to decide promptly which group of theorems contains the one needed for his purpose.

For example, it is required to prove that if a perpendicular is dropped from one end of a chord to the line tangent to the circle at its other end, the chord is the mean proportional between the diameter and the perpendicular. This proportion must depend on parallels or on similar figures; presumably, after examination of the figure, on similar triangles. The most promising way of attempting to prove the triangles similar, since the conclusion concerns lines and a circle is given, is by showing that two angles of one triangle are equal to two angles of the second triangle. The pupil draws the diameter from either end of the chord, so as to have it in position to be used with the other given lines, and examines the angles, beginning with the known right angle, and using the circle in measuring the angles. The other auxiliary line is now self-evident, and when this point in the analysis has been reached the proposition is practically solved.

The principal function of the teacher in this method is to be a leader of discussion, helping the class to avoid difficulties, guiding the investigation in such a way as not to discourage the expression of all the original ideas the class may have, and yet keeping them to the matter in hand so that time is not wasted in useless digressions. This does not mean that a wrong lead is not sometimes followed when it might seem to the pupils a natural method of attack; as a matter of fact, the following up of unusual suggestions often leads to proofs that are new to the teacher. This discovery of several proofs for the same proposition is one of the interesting features; as many as 11 proofs of one theorem, some of them almost if not quite unknown, have

been discovered by the pupils, some proofs with guidance from the teacher and others without it.

When a class is for the first time developing some new part of the subject, a very interesting recitation is sure to follow. A pleasure in discovery, or at the least in contributing some part to a discovery, is awakened, and the amount of new work that can be developed in one period and left to the class to put in finished form is, after the class is well grounded in the fundamentals and the methods of attack, an astonishing feature.

The beginning of the course is never hurried. Like any logical faculty, the geometrical sense is of slow growth, and a thorough understanding of the elements is so vital to progress that whatever time seems needed is given to the first book of geometry. After this part is well in hand the course moves on more rapidly, so that in the end time is actually saved. An average class can finish the subject, including a large number of exercises of various types and a great deal of drill on interdependence of theorems and on methods of attack, in a little more than three-quarters of the time allotted to it, thus leaving time for a thorough review of the subject matter and for plenty of practice on college entrance examination questions.

The success of this method of teaching geometry seems to be proved by the interest shown in the classes while such recitations as have been described are in progress, and by the increased logical strength of pupils so trained. It is not a cure-all, and should not be regarded as claiming impossibilities, for its excellence lies simply in the fact that it attempts to awaken and to train whatever logical faculties each student may possess. The relative ability of different students is not materially changed, but each pupil is encouraged and trained to use his logical power to as great an extent as he is capable, and each grows according to his potentialities.

For those who are interested in the details, I will add sample developments, the answers expected from the pupils being in parentheses, and explanations in brackets. Many of the questions and answers are in rather condensed form.

REVIEW LESSON.

Topic: *Proportions between sects.*

What is the most fundamental way of proving a proportion? (By the use of parallels cutting transversals.) What is the most important special case of this? (A line parallel to one side of a triangle.) What is the next way of proving a proportion? (By using corresponding parts of similar polygons.) What special case of this is of most importance? (Triangles.) When proving a proportion by triangles, which method of showing the triangles similar is most likely to be used? (By two angles equal.) Why? (Since the conclusion is to be a proportion between the sides, it is not as likely that such a proportion will be known at the start.) Is any other way possible? (Yes; two sides proportional and the included angles equal.) What will the new proportion be in that case? (The third sides proportional to a pair of those

used in the given proportion.) State three proportion theorems depending on similar triangles. [Different pupils are called on for the parts of the answer.] Outline the proof of each. [Pupils do so without figures, indicating the general method and the important steps. Questions are asked relative to the points likely to make trouble, and if the teacher is not certain that the pupils have visualized the figures and followed the proofs, the figures are then drawn on the board and discussed.]

DEVELOPMENT LESSON: NO THEOREM STATED.

Topic: *Trapezoids.*

Define trapezoids. Is anything additional known from the definition alone? (It has two pairs of supplemental angles.) [It is sometimes best to compare with parallelograms, which also have two pairs of supplemental angles.] We will use this recitation to find out all we can about trapezoids, and especially to see what theorems about triangles are also true of trapezoids. In order to do this, suggest a way to divide a trapezoid into figures you have already investigated. [Division into two triangles and into a triangle and a parallelogram are suggested; the triangle suggestion is found of little advantage at this stage, and the triangle and parallelogram division is decided upon.] What is known about the parts of the triangle formed? (The base of the triangle is the difference of the bases of the trapezoid, and the sides of the triangle are equal to the legs of the trapezoid; the base angles of the triangle are equal to the base angles on the longer base of the trapezoid.) State a theorem that applies to this triangle. (If two sides of a triangle are equal, the opposite angles are equal.) [Some other theorems may be suggested first, and can be used equally well if it can be extended to the trapezoid; if not, that fact can be shown.] Show that this theorem applies to the trapezoid. [A pupil does this when other theorems are called for, and the most familiar and important ones, such as the converse of the one already mentioned, and the unequal case, are stated and shown to apply to the trapezoid.] What about the angles on the shorter base? (Equal if the other pair are equal, and unequal in the opposite sense if they are unequal.) Why? (They have been shown to be supplemental to the base angles on the longer base.) What other lines in the trapezoid might we examine? (The diagonals.) Suppose an isosceles trapezoid with the diagonals drawn, what do you think is likely to be true of the diagonals? (They are equal.) What method of proof is most likely? (Corresponding parts of congruent triangles.) What triangles can be used? [Pupils name them and show them congruent.] Does this prove anything besides the diagonals equal? (The corresponding angles are equal.) Can these angles be used for any new fact? (The triangles from the ends of the bases to the intersection of the diagonals are isosceles.) [The recitation can be continued until every desired fact about trapezoids is found. The continuation of the legs to form a triangle, as well as the triangle formed by drawing a line from one vertex parallel to the other diagonal, might be investigated; the subject is prolific of simple facts, and is one of interest and of importance in relation to other figures. When it is finished, the student should have a good idea of the trapezoid and its properties. This is of course but one of many such investigations that are suited to a class discussion.]

DEVELOPMENT OF A PROPOSITION.

Statement: *To inscribe a regular decagon in a given circle.*

[This is chosen as a sample because it is one of the difficult propositions, and at first thought might not seem likely to develop readily. As a matter of fact, some pupils discover it practically without suggestion by applying the analysis method of attack.]

What is the best way to discover a construction? (Draw the required figure freehand, and by analysis try to find out how to construct it.) Suppose the circle O to be

given, and let AB be a side of the required decagon; what can you suggest? (Draw OA and OB.) How can you use the condition? (Angle O is one-fifth of a straight angle.) What else is known about the figure? (The triangle is isosceles, so angle A and angle B are each two-fifths of a straight angle.) What can be done to make use of the relative sizes of these angles? (Bisect angle B.) Why? (To form another angle equal to angle O.) Call the bisector BP; what is now known about the angles of the figure? [The pupils determine the size of each angle in terms of fifths of a straight angle.] What follows from this? (The small triangles are isosceles.) How many different lengths have the sects in the figure? (Three.) And what relation have the three? (The sum of two equals the third.) Which one is known and can therefore be used as a basis of length? (The radius.) Call it r; which do we wish to find? (The chord.) Call it x; how long is the third sect? ($PA=r-x$.) Have we made all possible use of the bisector BP when we use only the fact that it makes equal angles with the arms of angle B? (It also divides the line OA into parts proportional to the arms of angle B.) Give the proportions in terms of r and x. ($r/x=x/(r-x)$.) Are these sects all in one line? (Yes.) Then in what way is that line cut? (In mean and extreme ratio.) Then we have found that in a regular decagon the side is what? (The mean sect gotten by dividing the radius in mean and extreme ratio.) What would be the converse statement? (If the mean sect of the radius is used as a side of an inscribed polygon, the figure is a regular decagon.) But does the converse always hold? (No.) How do we find out whether it holds or not? (Take each "given," and see if the figures formed are the same.) In this case? (Take an angle equal to one-fifth of a straight angle, and a different chord equal to the mean sect, and see if each gives the other.) Do it. (The given angle has its chord equal to the mean sect by the proof just given: The chord taken is therefore equal to this chord, and must have an equal central angle; therefore the chord given equal to the mean sect has a central angle equal to one-fifth of a straight angle, and so is the side of a regular decagon.) What do you conclude to be the method of inscribing a regular decagon? (Cut the radius in mean and extreme ratio, and use the mean sect as a chord.) Why is it regular? (It is inscribed and equilateral.)

NOTE.—Soon after writing this development, one of my classes took up this proposition. I was much interested to see that the answers followed the plan given here almost exactly. If a stenographic report had been taken it could hardly have been more accurate, in content at least, than was this forecast of such a recitation. The only difference was that the class went on and found two other methods of proving the construction.

SAMPLE GEOMETRY TEST.

Given in February, 1911, to a beginning class that had studied rectilinear figures, including triangles, parallels, and parallelograms. The questions were not counted of equal value. Time allowed, one hour.

I. What is the sum of the interior angles of a polygon? Of the exterior angles? Prove one. Can there be a polygon such that the sum of its interior angles is five times the sum of its exterior angles? One such that the square of the number of straight angles in the sum of its interior angles is four times the number of straight angles in the sum of its exterior angles? Prove both answers, showing your method clearly.

II. What is the most fundamental way of proving triangles congruent? What is the principal use of congruence? List the ways of proving triangles congruent that use equal parts. How can corresponding parts of congruent triangles be recognized?

III. State three important facts about parallelograms, and prove one. What is a parallelogram, and how can this definition be used when a parallelogram is given?

IV. Prove that if two angles of a triangle are unequal, the opposite sides are also unequal, the —— (finish the statement.)

REAL APPLIED PROBLEMS.

[This part of the report was prepared by James F. Millis, of the Francis W. Parker School, Chicago, and relates to the work of that school.]

For a number of years the department of mathematics in the Francis W. Parker School, Chicago, has been experimenting along the line of teaching algebra and geometry in relation to their practical uses.

The Francis W. Parker School is primarily an experimental school. It was founded some years ago by Col. Francis W. Parker, whose name stands among those of the foremost leaders in the reform movements in education in the United States, for the purpose of putting to a practical test or carrying out certain fundamental ideas in the modern theory of education which he evolved or championed. In this democratic school each department has the greatest of freedom, both in regard to methods of teaching and to the organization of the content of the curriculum. The unifying influence in the work of the school consists of adherence to a few great fundamental principles in the modern theory of education, chief among them being that of self-activity of the pupil as a basis of all educative work, which demands that there be proper motivation of all school work and that the work shall relate to the experiences, interests, and needs of the individual pupil.

In conformity with these guiding principles, the aims of the mathematics work as planned in the high school are not merely those of mental discipline and preparation for college entrance, but it is the aim of the school that the student shall look upon the various mathematical subjects primarily as powerful and indispensable scientific instruments which the world has developed and uses in carrying on its practical work. Each mathematical subject has been developed, it is aimed to show, as a scientific body of principles and processes for use in solving certain types of problems that are actually encountered in the world's work. Algebra undertakes to solve problems of certain types, geometry problems of other types, calculus problems of still different types, etc.

Up to the present time the deviation in the work of the school from the traditional courses in algebra and geometry has consisted largely of two things: (1) Gathering together and using in the classrooms a large number of real applied problems of algebra and geometry that are encountered in ordinary everyday life in the various trades, in manufacturing, in the construction and use of various tools and measuring instruments, in science, in engineering, in architecture and designing, in carpentry, in sheet-metal work, in marine surveying, in navigation, and in many other fields of activity; (2) practical measurements, constructions, and field work.

In algebra a textbook has been used. But it has been supplemented by the use of mimeographed pages of practical problems. One important type of these problems consists of practical formulæ that are used in computation. These formulæ include, for example, those for computing the horsepower of engines, for computing wind pressure, for computing the draft in chimneys, for computing the deflections of beams, and a great variety of other such formulæ; simple electric formulæ, formulæ for computing the velocity of falling bodies, and other simple formulæ in elementary science, and the various formulæ used in practical mensuration.

The subject of algebra is approached through the use of these formulæ in making practical computations, beginning with those which express in symbols the simple rules of mensuration, interest, etc., with which the student is already familiar. Gradually and incidentally the idea of general number and literal notation, the use of exponents, signs of grouping, etc., are introduced as means of expressing and solving certain types of problems, and the student is in the midst of algebra before he knows it.

Similarly, the equation is developed as an instrument or device that is used in solving other types of problems. Enough real practical problems that are solved by equations have now been collected to supplant practically all of the traditional artificial problems that have filled textbooks.

In teaching geometry a very large collection of real problems has been made. Good applied problems of geometry are more easy to find than practical problems of algebra. These problems represent applications of all parts of elementary geometry, plane and solid. Many of them are problems of construction; many are problems requiring a proof; and others are problems of computation, based upon the theorems of geometry.

When the use of applied problems of geometry was first undertaken by the school they were mimeographed and placed in the hands of the students and used to supplement the "original" exercises of the textbook. With later classes the experiment was tried of teaching without a textbook, but placing in the students' hands mimeographed copies of a manuscript in which real applied problems were made an integral part of the subject. Finally, a textbook was evolved reorganizing the subject along new lines, emphasizing constructions, and, by incorporating the best of the applied problems that had been tried out, presenting geometry to the student, not as a mere exercise to be pursued for purposes of mental discipline alone, but as a scientific instrument for solving those problems encountered in the world's work for which it is adapted.

In addition to the use of real applied problems in the classroom, another phase of the work in geometry has consisted of practical constructions, measurements, and field work. For example, each pupil constructs on cardboard or metal a diagonal scale that will measure to hundredths of an inch. This is preserved and used in making accurate measurements in later problems or constructions. Its construction involves the problem of dividing a sect into any number of equal parts, and its use involves the principle of proportion in similar triangles. Students make an instrument called a quadrant with which they measure the altitude of the sun at different times of the year. The use of this instrument involves the principle that vertical angles are equal. The use of congruent figures is made in measuring the distances out of doors between points separated by obstacles. The principles of proportion are applied in determining heights and distances in the neighborhood. The plane table is used in measuring the distances between inaccessible objects, and in the construction of maps. This construction, measuring, and field work have proved of intense interest to students in the high school.

It is hoped to work out in a practical way, in the near future, the practical correlation of algebra and geometry with science and other subjects in the curriculum. A few points of contact of both algebra and geometry with physics, of algebra with chemistry, and of geometry with woodwork have been made. That much more of this correlation of algebra and geometry with other school subjects can be made is certain. For the student to see the subjects of algebra and geometry as scientific instruments that the world uses in its practical work is not sufficient; he should be able to use them to a considerable extent in the solution of practical problems arising in his own work in science, in the shops, etc. Problems that are the student's own problems are of the most intense interest, and it is in the solution of such problems that the greatest educational value lies.

The theoretical values of the use of these real applied problems in the mathematics work of the school are three. They lend real interest to the study of the subjects, and hence provide genuine, legitimate motivation of the work. They assist the student's knowledge to function by giving him practice while still in school in the use of that knowledge in the solution of problems such as he may encounter in later life. Finally, through their informational content a more adequate understanding of the environment in which the individual lives is afforded, and avenues of life interest opened that otherwise would remain closed to the individual throughout life. That the first of these values is a real one the school has already demonstrated.

A SECONDARY SCHOOL MATHEMATICS CLUB.

[This part of the report was prepared by C. W. Newhall, of the Shattuck School, Faribault, Minn., and relates to the work of a club in that school.]

The Shattuck School is a boarding school for boys of about 14 to 19 years of age. Its pupils come from many States, to be prepared for various colleges and engineering schools.

The mathematics club to be described was organized in 1903. Its membership is limited to the students of the senior class in mathematics, who have finished the usual course in plane geometry, have had two years' work in algebra, and are studying solid geometry, trigonometry, and advanced algebra. The instructor of the class acts as leader of the club and presides at the meetings.

The meetings are held on the evening of the weekly holiday, and, as an offset to the time devoted to the meeting, no preparation is required for the regular classwork in mathematics for the next day. Each of the 15 or 20 members is expected to prepare a report (requiring about 10 minutes for presentation) once in 5 or 6 weeks.

The object of the club is to study unusual and interesting mathematical topics which do not find a place in the regular curriculum. The subjects include topics from the history of mathematics, famous problems, puzzles, fallacies, and tricks—anything, in fact, which is capable of a mathematical solution or explanation, and which promises to be interesting. This last criterion is most important, for a voluntary club of school boys will not thrive if the meetings are dull. The program of last year, giving a list of the subjects discussed, is appended to this statement.

As the boys have had no previous experience in such work, they need help in the preparation of their reports. Brief outlines are furnished, giving rather explicit suggestions as to the treatment of the topic, what to include, and especially what to leave out. For historical topics it is easy to refer the student to books, but for such a subject as "Mathematical Symmetry in Nature" he must hunt more widely for his material and needs more suggestions. In taking up "Non-Euclidean Geometry," "Infinity," and "The Fourth Dimension" the leader must point the way with still more care, and with such a subject as the "Foundations of Geometry," he must open the discussion and perhaps do the larger part of the work himself.

A list of the books in the mathematical library of the school, which have been found most useful in the club work, is appended. A collection of some 40 or 50 articles on appropriate subjects taken from popular magazines is also available.

At each meeting three or four special topics are presented, all bearing on a single general subject. The special topics as well as the general subject for each meeting are listed in the program for 1909–10 referred to above. There is no prescribed form of presenta-

tion. Some of the boys speak from notes, referring perhaps to illustrations previously placed on the blackboard, others prefer to read selections directly from the books which they have consulted, while still others prepare carefully written papers which in some cases have been subsequently elaborated into graduation theses.

Each report is followed by informal discussions during which the speaker is expected to answer any questions which may be raised. The leader may come to his assistance in case of need or may add a brief comment upon the topic, but one of the cardinal principles of the club is that the boys shall do most of the talking.

For the purpose of giving some suggestion of the spirit of the meetings, we may describe in some detail the treatment of a subject which always arouses the keenest interest—puzzles and fallacies.

The subject is presented by first placing a fallacy or puzzle before the members and giving them a reasonable time for exercising their wits in its solution. At the end of the allotted period, the trick is "given away" if not already discovered, and the mathematical principle is explained.

An idea of the character of the fallacies discussed may be had by consulting, for geometric fallacies, Ball's, White's, or Ozanam's books on Mathematical Recreations; and for algebraic fallacies, Viola's Mathematical Sophisms. The puzzles include the innumerable card tricks which rest on some mathematical formula or some principles of arrangement and tricks by which the performer discovers one's age or birthday, the number of spots on a card, etc., by requiring one to perform a series of arithmetical operations whose result reveals the required number. The following example of this type was given at one of the meetings:

The performer chose seven other boys and, during his absence from the room, a ring was placed upon a certain joint of a certain finger of a certain hand of a certain boy. The right and left hands were numbered 1 and 2, and joints, fingers, and boys were also designated by number. The four numbers indicating the position of the ring may be designated by u, v, x, and y.

On returning to the room, the performer called for certain multiplications and additions of these four numbers resulting in the expression $1000u+100v+10x+y$. He then called for the result and, on being told that it was *4132*, announced that the ring was upon the second joint of the third finger of Smith's right hand.

In conclusion, I may say that this mathematics club has proved to be a most interesting and valuable adjunct of the regular work of the classroom.

The members, to be sure, acquire no very profound knowledge of the subjects discussed but they do come to have a broader view of the scope of mathematics and—the most valuable result—they are stimulated to independent thinking.

PROGRAM FOR 1909-10.

First Meeting.—Algebraic Fallacies.

An informal consideration of certain proofs (?) that 2 equals 1, 1 equals 0, 1 equals −1, etc.

Second Meeting.—Our Number System.

1. First notions of numbers.
2. Primitive numeration.
3. Development of decimal system.
4. The positional idea.

Third Meeting.—Number Systems and Symbols.

1. History of our Arabic symbols.
2. Number symbols of other systems.
3. Nondecimal systems.
4. Some problems in a nondecimal system.
5. The duodecimal vs. the decimal.

Fourth Meeting.—History of Arithmetic and Algebra.

1. Among the ancient nations.
2. Among the Greeks and Romans.
3. Among the Hindus and Arabs.
4. In Mediæval Europe.
5. The development of algebraic symbolism.

Fifth Meeting.—Numerical Curiosities.

1. Mystic properties of numbers.
2. Prime numbers, triangular numbers, squares, cubes, etc.
3. Magic squares.
4. Large numbers.
5. Number forms.

Sixth Meeting.—Numerical Curiosities (continued).

1. The number 9 and its properties.
2. Other curious numbers.
3. Mathematical short cuts.
4. Mental calculations.

Seventh Meeting.—Numerical Tricks and Puzzles.

1. Numerical tricks.
2. Numerical puzzles and catch questions.
3. To discover a number thought of.

Eighth Meeting.—Geometrical Tricks and Puzzles and Mathematical Games. Informal consideration; no formal reports.

Ninth Meeting.—Card Tricks involving some mathematical principle of number or position. Problems on a chess board.

Tenth Meeting.—Foundations of Geometry.

1. The assumptions.
2. Nature of space.
3. Definitions.
4. Logic of geometry.

Eleventh Meeting.—History of Geometry.

1. Beginnings of geometry.
2. Early Greek geometry.
3. Euclid and his immortal elements.
4. Recent developments in geometry.

Twelfth Meeting.—Famous Problems of Geometry.

1. Squaring the circle.
2. The duplication of the cube.
3. Regular polygons and polyhedrons.
4. Famous problems of solid geometry.

Thirteenth Meeting.—Foundations of Geometry.

1. Geometric assumptions.
2. The straight line, and how to draw one.
3. Non-Euclidean geometry.
4. Some criticisms of the class textbook.

Fourteenth Meeting.—The Mathematics of Common Things.

1. The mathematical principles of maps.
2. Optical illusions.
3. The carpenter's square.
4. Weighing and measuring.
5. Mathematical symmetry in nature.

Fifteenth Meeting.—The Fairyland of Mathematics.

1. The fourth dimension.
2. A visit to Flatland.
3. A visit to Infinity.
4. Curved space.

Sixteenth Meeting.—Higher Mathematics.

1. History of trigonometry.
2. History of logarithms.
3. Calculus, and other pleasures to come.
4. A world without mathematics.

REFERENCE BOOKS.

Below are appended the titles of a few books and pamphlets which have proved most valuable in the work of the club. Only publications in the English language are mentioned. Additional references will be found in Smith's "Teaching of Elementary Mathematics," Young's "Teaching of Mathematics," Withers's "Parallel Postulate," White's "Scrap Book of Elementary Mathematics," Ahrens's "Unterhaltungen und Spiele," etc.

This list does not include textbooks, nor articles in encyclopædias, magazines, etc. Among magazines frequently containing such articles of interest are School Science and Mathematics, The Open Court, The Monist, Science, The Popular Science Monthly, The Scientific American, etc.

(1) Foundations and criticisms.

Common Sense of the Exact Sciences	Clifford	Appleton.
Space and Geometry	Mach	Open Court.
Euclid's Parallel Postulate	Withers	Open Court.
Euclid	Frankland	Wessel & Co.
Non-Euclidean Geometry	Manning	Ginn & Co.
Geometric Axioms (Popular Science Lectures, 2d series).	Helmholtz	Appleton.
The Thirteen Books of Euclid's Elements, with Introduction and Commentary.	Heath	Cambridge University Press.
Mathematical Monographs	Young	Longmans.
Lectures on Elementary Mathematics	Lagrange	Open Court.
Theories of Parallelism	Frankland	Cambridge Press.
Number Systems	Fine	Ginn & Co.
The Teaching of Geometry	Smith	Ginn & Co.

(2) *History.*

History of Mathematics	Cajori	Macmillan.
History of Elementary Mathematics	Cajori	Macmillan.
Short History of Mathematics	Ball	Macmillan.
Primer of the History of Mathematics	Ball	Macmillan.
Greek Geometry from Thales to Euclid	Allman	Dublin University Press.
History of Greek Mathematics	Gow	Cambridge University Press.
Euclid	Smith	Scribner.
The Story of Euclid	Frankland	Wessel & Co.
Famous Problems of Geometry	Klein	Ginn & Co.
Mathematical Monographs		Heath & Co.
History of Teaching of Elementary Geometry	Stamper	Teachers College, Columbia University.
Portraits of Mathematicians	Smith	Open Court.
The Teaching of Elementary Mathematics	Smith	Macmillan.
Rara Arithmetica	Smith	Ginn & Co.
Sixteenth Century Arithmetic	Jackson	Teachers College, Columbia University.
The Hindu-Arabic Numerals	Smith and Karpinski.	Ginn & Co.

(3) *Recreations.*

Scrap Book of Elementary Mathematics	White	Open Court.
Mathematical Recreations	Ball	Macmillan.
Mathematical Essays	Schubert	Open Court.
The Canterbury Puzzles	Dudeney	Dutton & Co.
Paradoxes of Nature and Science	Harpson	Dutton & Co.
The Number Concept	Conant	Macmillan.
Philosophy of Arithmetic	Brooks	Normal Publishing Co., Philadelphia.
Recreations in Science and Mathematics [1]	Ozanam	
Scientific Romances	Hinton	Swan, Sonnenschein & Co.
Fourth Dimension	Hinton	Swan, Sonnenschein & Co.
Flatland	Anon	Little, Brown & Co.
Geometric Exercises in Paper Folding	Row	Open Court.
How to Draw a Straight Line	Kempe	Macmillan.
Geometry and Faith	Hill	Lee & Shepard.
Short Cuts and By Ways in Mathematics	Blakie	
Visit to Algebra Land	Ward	Educational Publishing Co., Syracuse.
Primitive Culture, Vol. I	Tylor	Henry Holt.
Magic Squares and Cubes	Andrews	Open Court.
The Fourth Dimension	Manning	Munn & Co.
Pleasure with Profit	Leybourn (1693)	Out of print.

[1] Translated by Hutton; out of print; only second-hand copies obtainable.

CONCLUSION.

The preliminary classification of schools has been used comparatively little in describing courses of study and methods. Although location, religious connection, and various features of organization were considered in making tabulations, few distinctions between these various types of schools appeared in the data bearing upon the teaching of mathematics. The classification, while unnecessary for describing differentiation in courses and methods, has been retained because of its value in a description of the general characteristics of the schools included in the report.

The data reported by the schools have been presented as accurately as possible, with almost no comment. The report is not a complete description of mathematics teaching in private secondary schools, and we have indicated roughly limitations to the reliability of the figures, but in this form the report should be of more value for comparison of the schools of this field with the others than if interpretation reflecting the personal convictions of the members of the committee had been given.

In the discussion of aims, however, criticism of the prevailing views with regard to certain questions is perhaps implied, and in closing the report a word of commentary may be permitted. The statement in regard to coeducation and to the various questions of aim show that few teachers are in the habit of analyzing their problem carefully. One of the greatest needs of mathematical education in secondary schools to-day is the scientific determination of its legitimate purposes. Teachers should be less content to be guided by an examination requirement or to have blind faith in the supreme value of their subject. There should be a conscious purpose in all teaching and frequent attempts to measure results.

APPENDIX.

The following reports relate to institutions which, though not secondary institutions exclusively, cover more or less of the secondary field in their work.

A. MATHEMATICAL INSTRUCTION FOR EVENING TECHNICAL SCHOOLS.

Meeting requirements of compulsory education.—With the exception of some privately endowed schools, the evening schools have in the past made but little attempt to provide mathematical instruction of a sort that would meet the requirements of men and boys engaged in our constructive industries. In fact, most of the instruction in mathematics has been of an elementary order, meeting only the various legal requirements in the several States with reference to employment certificates. For the benefit of those who are unfamiliar with the spirit of such laws, the following extract from the education law of New York State is given:

> Every boy between 14 and 16 years of age * * * who is engaged in any useful employment or service in a city of the first class or a city of the second class and who has not completed such course of study as is required for graduation from the elementary public schools of such city, and who does not hold either a certificate of graduation from the public elementary school or the pre-academic certificate issued by the regents of the university of the State of New York or the certificate of the completion of an elementary school issued by the education department, shall attend the public evening schools of such city, or other evening schools offering an equivalent course of instruction, for not less than six hours each week for a period of not less than 16 weeks in each school year or calendar year.

The intent of all such laws has been to provide for the illiterates. Consequently evening classes have been filled with boys and men of all ages, interests, and capacities, who could hardly understand the language of the classroom and who knew even less about the simplest operations of arithmetic.

Such a procedure naturally developed a series of elementary evening schools with little or no plan beyond that of meeting the State laws. Furthermore, the pupils themselves were practically driven to these schools by officials appointed to enforce compulsory attendance.

Development of evening high schools.—A series of evening high schools has gradually been developing in the larger cities. These

schools have reached a higher class of pupils—young men and women who could do more than read and write.

At first, many of them were naturally studious and sought with definite purpose educational advantages in the lines of algebra, geometry, and trigonometry. Many were preparing for entrance to some college while at the same time they were earning a living.

Later the commercial courses were introduced. They appealed to a much larger constituency, who were perhaps not naturally studious and yet who sought definite instruction in commercial arithmetic, bookkeeping, etc.

Within the last 10 years the shop and laboratory courses of our manual-training and technical high schools have been made available to the pupils of the evening schools. Thus we find another broadening out of evening instruction, meeting the needs of boys and men working in manufacturing industries.

Present weakness.—We note how the public evening schools have broadened their scope so far as educational activities are concerned. The same can not be said with reference to the method of instruction pursued, for in too many instances it has been haphazard, indefinite, and unorganized. Day-school teachers, unfamiliar with the special needs of evening students, have been engaged; textbooks written for the day elementary and high schools have been adopted; methods of teaching peculiar to the requirements of the immature day-school pupil have been thrust upon the mature members of the evening classes.

Privately endowed schools.—Fortunately, throughout the period of the development of the public evening school which has been briefly described, certain institutions—notably, Cooper Union, in New York City—have been steadily building up a comprehensive scheme of evening instruction which definitely meets the individual requirements of those engaged in technical work. Courses of instruction covering a definite period of years, trained teachers, special notes, and general improvement based upon previous experience, have made these schools leaders in the field of evening technical instruction.

Main purpose of the report.—The movement toward better public evening school instruction along lines of technology is so recent and the number of privately endowed evening schools of the same order so few that this report is necessarily limited in its statement of any definite development of high grade institution. Perhaps, after all, it can serve no more useful purpose than to outline some fundamental points which must be considered if such instruction is to be more effective, particularly in view of the fact that these points have been suggested by the answers to inquiries sent out by the committee.

Form of blank.—A series of questions was sent by the committee to all evening trade, technical, and industrial schools, covering the following points:

1. Nature of school—academic, or technical, or trade school.
2. What degrees granted, if any; and for how much work.
3. Number of years required for full course.
4. Approximate number entering each year.
5. Approximate number graduated each year.
6. Nature of course in mathematics. Is mathematics taught with relation to the students' occupations?
7. Number of hours per week devoted to the various mathematical subjects.
8. What textbooks are used?
9. Are these satisfactory; or if not, in what points are they defective?
10. In what way could the textbooks you use be bettered, if any?
11. What is the business of the night instructors during the daytime; that is, is it at all related to those subjects which they teach at night?
12. What is the average proportion of your instructor's salary for his evening work to his day salary?
13. In what way could the "personnel" of your instructing staff be bettered, if any?
14. What proportionate amount of time should be given, in your opinion, to the various branches of mathematics?

The following is a summary of the replies to the more important of these questions:

Improvement of evening schools.—An important educational move in the immediate future should be in the direction of improving the instruction in evening schools and adapting them to the needs of industrial workers. The methods of these schools should be recast. They should adapt themselves to modern industrial conditions, and through proper instruction of practical subjects touch more closely the economic and social life of the times. The evening school student attends to satisfy a definite need. These students have already received a more or less formal education in the public schools. They are receiving in their daily work incidental industrial experience, and have learned from this thorough teacher that they are deficient in some lines; hence this endeavor, outside of their working hours, to fit themselves for definite lines of activity.

Vital needs.—There are certain vital needs in the organization and methods of conducting evening industrial improvement schools. The evening technical school deals with two rather distinct classes: First, those who are naturally students and seek with a definite purpose educational advantages in the advanced lines of mathematics, and

mechanics; second, those who are not naturally students and yet who seek with a more or less definite aim educational help in a solution of some present problem which involves special service. The latter class is interested in shop mathematics, elementary engineering practice, etc.

Courses must be of two kinds.—The recognition of these two classes means that the courses of instruction must be of two kinds, one comparing favorably with the day-school work in its general scheme, the other and major part differing decidedly from the methods ordinarily pursued. The evening work of the nonstudent class must have its own distinct ideals, methods, and estimates of value based upon current community conditions and individual needs rather than based on the regular school standards which are applicable primarily to the student class.

Teaching staff.—Day-school teachers are employed too much at present in evening schools. These teachers can meet the needs of the student class, but they can not properly teach the nonstudent class which often consists of industrial workers. To the custom of employing day-school teachers must be laid much of the lack of definiteness in the planning of evening-school work. It is a very simple matter for the average day-school teacher to adopt the regular textbooks and to use the regular outlines and methods. This is a perfectly consistent action, for few regular teachers have opportunity to know the vital industrial needs of their students through their own academic experience. Now, the only people competent to teach in our evening industrial and technical schools, even on the book side, are the men and women who know from their contact with modern industrial and commercial life vital points of interest which concern these workers who come to the evening schools to meet definite needs.

A few typical answers to question No. 11 will show how few instructors are in touch with the practical aspects of technical problems relating to their mathematical teaching.

"Three teach mathematics during the day." "We employ day-school teachers." "Practically all our teachers teach the same subjects in the day school." "Both are teachers in the day school." "Teach mathematics in the daytime." "Teacher in grammar school." "All teach in day schools."

Contrasted with these statements are those from the majority of the evening technical institutes like Franklin, Cooper, and Lewis.

"Only two of our day-school teachers teach in evening school." "Night instructors are employed in engineering work during the day." "Part are teachers; remaining ones are engineers and draftsmen." "Most of our teachers have been with us for many years and are thoroughly capable."

The direct appeal.—Evening school instruction must appeal to the student immediately at the beginning of his work. The subject matter of the early lessons must satisfy his need as he has defined it. The success of evening instruction depends upon this principle. For example, a young machinist has received a reprimand from his foreman because he can not estimate the amount of "set-off" of the tailstock of a lathe to cut a taper. He enrolls in a class in trigonometry to meet that deficiency and finds that the first two lessons are concerned with definitions, the next three with quadrants and tables, and that the remainder of the term is to be spent on the development of formula. During this time he is receiving in his daily work the same reprimands, and is therefore debating in his own mind the value of his evening instruction. The average apprentice machinist does not see the direct application of this instruction to his work. He enrolled for a definite purpose. It would have been perfectly possible to give in the first five lessons some elementary but practical instruction in the application of the right-angled triangle to the offset of a tailstock. Instead of leaving school at the end of the fifth lesson with no instruction which appealed to him, he would have received enough in those five lessons to fit him to meet the demands of his foreman and more than likely he would have remained in the mathematical class to receive the more definite and thorough instruction in its theory which must be gained if one is fully to comprehend and cover the entire range of the subject.

Flexible courses.—The various features of the different courses in mathematics must be elective and flexible and presented in small and varied units. Instead of printing in a course of study "arithmetic," "geometry," etc., there should be printed, "arithmetic for mechanics," "arithmetic for clerks," "mechanical drawing for apprentices," etc. Where it is possible even a finer differentiation is desirable, such as "arithmetic for plumbers," "arithmetic for errand boys," "geometry for machinists," etc. Not only will this presentation serve to catch the eye of the prospective student, but it will also suggest to him that special effort is to be made in the class work to help him in his daily occupation. The instruction in the various branches must be adapted to the needs of the various occupations. The terms used in the classroom must savor of the shop, office, and store. Experience shows that the problem "What is ⅔ of 37½?" does not appeal so much to a clerk as the problem, "What will ⅔ of a yard of cloth cost at 37½ cents a yard?" On the other hand, the latter problem does not awaken the interest of the mechanic as much as the problem involving the same operations, which reads, "If a copper casting weighs 37½ pounds and specific gravity of iron is ⅔ that of copper, what will the casting weigh if made of iron?"

Departmental system is not always suited.—Oftentimes the students will do better work and more will be accomplished if the mathematical instruction is definitely related to the vocational work and is given by or under the direction of the teacher of the technical subject. For example, it is possible to teach the mathematics of steam engineering in connection with steam-engine practice. If the student sees the need of using a certain formula he can be better taught the derivation of that formula when the necessity arises for its use. In general, it may be said that the major subject in evening-school work is the vocational one—that is, the one related to the daily occupation of the student—and that all the other subjects, such as mathematics and science, are but incidents to the accomplishment of the vocational purpose. An illustration will serve to make clear: A machinist enrolls in an evening school for mechanical drawing and finds that he needs to brush up in fractions and decimals and that he needs square-root in order to work out a formula for screw threads. The opportune time to teach him these topics is when the need for them arises, and none is more qualified to give the required practical instruction in such topics than a competent drawing teacher. The value of sending him to another class for a line of mathematics unrelated to his main "center of interest" is questioned.

Along the same general lines the following description of work in Lewis Institute, Chicago, may be cited:

The larger group, however, is composed of men and boys who must have a practical grasp of mathematical principles for their work in engineering. Here there is no attempt to teach mathematics as a thing apart from its applications. For the first 20 weeks the students in the first-year engineering principles, about 250 in number, meet two evenings a week for two hours. During the first hour in the mathematics lecture room a sheet of problems is given to each student, and the instructor works the problems on the blackboard and explains the principles involved. These problems are based directly on the second hour's work, which consists of a demonstration lecture in applied physics. Thus, in the physics lecture room a crane is set up, and spring balances are arranged to show the stresses in the boom and tie. The following evening in the mathematics lecture room problems on the crane and triangle of forces are solved and the mathematical principles are discussed. A motor-power test with the Prony brake is the basis for work and power problems. During the third term of 10 weeks the students spend two evenings a week in the laboratory and one evening in solving problems based on their laboratory work. At the end of the year each student has a set of sheets containing the necessary definitions, tables, and statement of principles from the physics lectures, a set of laboratory report blanks in which is a record of his laboratory work, and a set of problem sheets with a solution of the problems. These are bound for the students, and by this means they have a permanent record of the year's work for future reference and study.

During the remaining three years of the engineering work the mathematics is continued in the closest possible relations with it. All the while the methods of solving the various problems that arise are kept as simple as possible. A judicious use of arithmetic, a very little algebra, or a simple diagram secures results that are often obtained through elaborate processes involving lengthy operations and complicated equations. Many of the students learn to use a table of four-place logarithms, the

slide rule, and the table of series, cosines, and tangents skillfully and intelligently. We have learned that it is not necessary to tell a man that he can not enter an engineering class in the evening school till he has studied mathematics one or two years. He can get a working knowledge of mathematics through the problems.

In the Springfield (Mass.) Evening School of Trades the instructor of drawing used a method similar to the following:

When the student reached a place in a drafting course dealing with the subject of screw threads, it became necessary for him to apply some such formula as $P=0.24\sqrt{d+0.625}-0.175$, where P is the pitch of the thread and d is the diameter of the bolt. This problem involves square root and decimals. One hour of individual or small-group instruction by this teacher gave a student the necessary familiarity with these mathematical processes to make them sufficiently clear to him in their application to the formula.

Undoubtedly many students are not satisfied with this incomplete instruction, and this is often made evident through their joining the regular class in mathematics the next year in order to gain an insight into the reasons involved in the process of square root. Instead of thorough preparation in mathematics for mechanical drawing, it would be better to have the mechanical drawing lead the students into mathematics. This reversal of the usual procedure, while it may not be pedagogical so far as the subject matter is concerned, is certainly true to experience when one deals with the characteristics of the average evening-school student. The teaching of application before theory should be always emphasized in evening instruction.

Sequential arrangement of courses.—Mathematical instruction should have a sequential arrangement, elementary and advanced courses being given. Experience teaches that evening schools are so overcrowded in the elementary courses that the more advanced students suffer through insufficient attention. If the student's transient need is well met, it will place him in a better position, only again to make him feel a renewed need of self-improvement. This means that he will return to the evening school in some subsequent year, when he ought to be given advanced work. Not only must each school year's work be driven home and clinched, but each series of years' work must be so clinched as to meet the needs of industry. The strength of the work of such evening institutes as Cooper, Pratt, Lewis, and Franklin testifies to the value of sequential arrangement of courses.

Classification by ages.—Where possible there should be in the evening-school work a separation in the class instruction of the immature from the mature nonstudent class. The latter are extremely self-conscious. Their feelings should be respected as far as possible. Is it any wonder that a foreman of a pattern shop does not take kindly to being instructed in arithmetic in the same class with a boy machine tender over whom he has charge during the day?

Classification by vocations.—Undoubtedly, as far as is possible, students should be classified in the mathematical classes according to their trade or business. To-day workingmen have common trade interests. Evening-school students grouped according to occupation would have an opportunity to talk over these interests. The teacher

could act as a leader and draw out the students into telling their trade experiences, and through the expression of these various opinions the most practical solution of the particular problem at hand would be obtained. Teachers who have had evening-school experience know how difficult it is to get the students to recite and express themselves at the blackboard. A free discussion of the point at issue makes the student lose his self-consciousness and before he is aware of it he is at the board illustrating his particular method of solution.

It is recognized by the committee that small schools will have difficulty in dividing the classes into occupational groups. However, any evening school with an enrollment which requires the formation of more than one class in a given subject can at least divide its class enrollment into two divisions: (1) Those engaged in mechanical, electrical, and steam-engine practice trades during the day; (2) those engaged in building trades during the day. The larger the enrollment the finer the differentiation as based upon the daily occupation of the students.

Textbooks for industrial workers.—Textbooks written expressly for the kind of instruction demanded in evening schools should be used. There should be a marked difference between the methods of instruction in day and evening schools. What is needed, for instance, is not an elaborate textbook in general arithmetic, with all its topics of fractions, decimals, square root, percentage, interest, partial payments, bank discounts, etc., but rather a book which appeals to a man in the machine trades, then one which appeals to a plumber, or a clerk, or an errand boy; small enough to slip into the side pocket of a coat and cheap enough so that he can readily own a copy for reference in his daily work.

Furthermore, there should be textbooks so arranged as to relate a series of questions to one given plan; it would be much better than a large number of totally unrelated questions. In addition it may be said that those questions should deal with simple actual problems.

B. THE TEACHING OF MATHEMATICS IN PRIVATE CORRESPONDENCE SCHOOLS.

Preliminary statement.—In the preparation of this report the most complete list of correspondence schools possible was obtained. The list showed that many of these institutions were of such nature that they did not give instruction in mathematics. In obtaining the data for the report the committee sent a blank which contained questions asking for the information desired by the general committee. One hundred and thirty-five correspondence schools, colleges, and universities doing correspondence work received these inquiries. Only a

small number replied. For this reason the conclusions in this report are drawn from insufficient data to make them of the highest value. Nevertheless, the replies received were from such institutions as make them fairly representative of the field covered by correspondence instruction in mathematics.

Historical sketch.—Instruction by correspondence is of comparatively recent origin. It was first successfully developed in Germany about the middle of the nineteenth century. The first attempts to introduce this method of instruction in the United States were in connection with the Chautauqua Literary and Scientific Circle, which organized a department of correspondence in 1884. Since then the work has extended throughout the country, and the correspondence method of instruction is employed not only by independent schools, but also by a number of the leading colleges and universities.

Two classes of institutions are engaged in instruction by correspondence. These are the independent correspondence schools—institutions doing all of their work by correspondence—and colleges and universities having correspondence departments. The following report is divided into two parts, the first part dealing with the independent correspondence schools and the second with correspondence departments of other institutions.

Independent Correspondence Schools.

Organization.—The plan of organization of independent correspondence schools varies somewhat in detail, but in their essential features all plans are nearly the same. No system of classes or class work exists, and instruction is with the individual student. With scarcely an exception these schools are entirely independent of other institutions of learning. However, those whose instruction is recognized as thorough often have their students accredited in higher institutions of learning which the said students desire to enter. This credit, it should be said, is a matter resting entirely with the accrediting school, and unless the student so accredited is able to sustain himself in the department which he enters, the credit is withdrawn. Nearly all of the best schools have courses which prepare their students for the freshman year of colleges and universities, but most of them are also highly specialized and devote the greater part of their time to preparing their students for the various vocations which these students wish to follow. For this reason the courses in mathematics and the instruction given are on a much more limited plan than courses of equal strength in high schools, colleges, and universities.

Courses of study.—The reports received show that arithmetic, algebra, and geometry are taught in a large majority of those schools which teach mathematics at all. A limited number have courses in

trigonometry and commercial arithmetic, and a still smaller number have special courses, by which is meant courses especially adapted to fitting the student for some vocation, such as accounting and electrical engineering. While the schools offering such courses are few in number, their patronage is very large, and most of the courses in mathematics which these schools offer are devoted to the various lines of industrial work.

Methods of instruction.—The method of instruction is practically the same in all of these institutions. Texts are sent the students and they are required to prepare the lessons assigned on these texts. When the lesson is prepared the student writes his lesson paper, answering questions and solving problems required by the school. This paper is forwarded to the school, where it is reviewed, criticised, and usually graded; then returned to the student, together with such suggestions and directions as the instructor feels that the student needs.

The plan of sending out the lessons varies. Some schools send at once a volume containing all the lessons of the course; others send pamphlets containing one or two lessons each, and when the course is completed, present the student with a bound volume containing all the lessons previously sent in pamphlet form. The questions for the lesson paper may be in the text or they may be sent from time to time as the student requires them. The schools vary also in their plan of assisting students. Some schools send along with the first lessons carefully prepared directions for study, and others give these directions from time to time as the lesson papers are returned. In every case the student is thrown upon his own resources in the preparation of his lesson and his lesson paper.

Textbooks.—In the matter of supplying students with textbooks three plans are followed. Some schools use a special text, prepared by their own instructors and adapted to the needs of their students. Others use the regular texts employed in residence work, sending the students questions based upon the lessons in these texts. The third plan uses the regular textbook and supplements it by a correspondence instruction book, in which the lessons are outlined, instructions for study given, and occasionally supplementary exercises added. These instruction books in most cases also contain the questions upon which the lesson paper is based.

The special texts used in correspondence instruction are prepared as carefully as those used in residence work. Nevertheless, these same books differ widely from the general textbooks on the same subject. Concerning this the report from one of the schools makes the following statement:

The aim of the author of the ordinary textbook is to produce a work that may be used by all who wish information that would naturally come under the heading under which the book would be classified, and he is not at liberty to restrain the scope of his book by leaving out sections which are ordinarily included in works of that character.

The leading correspondence schools, however, are constantly and deliberately violating all recognized rules of textbook making. They aim to give the student exactly what he wants and needs in connection with the particular line of study he desires to pursue, and to give him no more and no less, and the books are prepared in such a manner that the student can obtain the information desired in the shortest possible time.

Furthermore, the makers of these books aim to make them so clear that they can not be misunderstood by one of average intelligence, and to make all explanations so complete that the student will have no trouble in interpreting them for himself. In the correspondence textbook, in other words, all the difficulties of the student are as far as possible anticipated, because the author keeps constantly in mind the fact that many of these students can not get assistance from any one except by writing to the school, and in some instances it may take several weeks or even months to obtain an answer.

Another important particular in which the correspondence text varies from that for residence work is in the space it devotes to those particular phases of the subject needed by the student; that is, the arithmetic for use in a course of engineering will contain much more upon the subject of evolution than is found in the ordinary arithmetic of similar grade, while it may contain little or nothing on the subject of denominate numbers or of percentage.

The textbooks in mathematics prepared by correspondence schools are particularly "doing" books. They show the student how to arrive at the results he must attain in the shortest and most practical way, omitting the discussion of principles or the reasons underlying the formula and rules given. They prepare the student to perform some special line of work, but in gaining this preparation he acquires little or no power to solve problems in other lines of work, unless they are very closely related to the course which he has studied.

Conclusion.—The committee found no independent correspondence schools which have special courses in mathematics for the preparation of teachers of this subject. Some of them offer review courses with a view to assisting the teachers, but these are limited in extent and apparently contain but little in the way of directions for teaching.

In the matter of examinations the committee found no uniformity whatever. Some schools require examinations at stated intervals; some when the course is completed, and others consider each lesson paper an examination, and when the last paper is received consider the course to be completed. If the student has met the required standard a certificate to this effect is returned to him with his last paper. Where examinations are given there is also a wide difference in the value assigned them in determining the student's standing at the completion of his course. Some schools consider them to be of equal value to lesson papers; others consider the final examination to have two-fifths of the value of the recitation papers and few place but little value upon them. From the reports concerning examinations the committee was unable to draw any satisfactory conclusion. Considering the reports as a whole, it would seem that less attention was paid to written examinations in correspondence work than is usually assigned that subject in residence work.

Universities and Colleges.

Organization.—All colleges and universities reporting to the committee state that their correspondence work is organized as a department of the institution. These departments have no direct connection with other schools, but without exception the credits gained through correspondence work with colleges and universities are accepted not only by the institution giving the course, but by any other university, college, normal school, or high school to which the student applies for admission.

Courses of study.—Most of these institutions offer courses in correspondence parallel with those offered in residence work. One replies: "We offer courses in elementary algebra covering one year of the usual high school course; plane geometry covering one year of high school work; solid geometry representing the usual half year of high school; higher algebra representing the half year of advanced high school work; higher algebra representing one semester of freshman work in a university and trigonometry covering plane and spherical trigonometry and representing one semester of freshman work." One of these, a State University which has established a very extensive correspondence department, offers 12 courses of elementary grade and 6 courses of university grade. Six of the elementary courses are vocational, each being designed especially for the purpose of assisting those who take it in the particular vocation they are following. In this respect these courses and the texts containing them follow very closely the plan of similar courses prepared by independent correspondence schools doing industrial work. The elementary courses and those of university grade in this institution are similar to the same courses provided for residence work.

Methods of instruction.—Methods of instruction in vogue in the correspondence departments of colleges and universities are practically the same as those employed by independent correspondence schools. However, a larger proportion of these institutions use the ordinary textbook, expecting their students to prepare their lessons from these texts as best they can. When these texts are used, questions upon which the lesson papers are based are sent the students at regular intervals. Some of these schools send students special instructions with the return of their lessons or with the assignment of lessons.

Preparation of teachers.—From the replies received it appears that a majority of colleges and universities maintaining correspondence departments provide through these departments for the training of teachers in mathematics to a limited extent. An occasional institution gives a special course for this purpose, but the others give the work in connection with an ordinary course in the subject studied, as in arithmetic or algebra.

The requirements for admission to these courses vary somewhat widely. Some institutions will not allow undergraduate students to take teachers' courses. Others open these courses to those who have completed a four-year high school course and two years' work at a good university or college. Some of these institutions require the candidate to enter upon a course leading to the degree of Ph. D. and to do a part of the work in residence, devoting at least one-third of his time to giving instruction in undergraduate classes. Another institution in addition to the work in mathematics requires the study of the pedagogy of mathematics, a course which concerns itself chiefly with reasons and methods for teaching elementary mathematics, especially arithmetic.

Conclusion.—The replies received by the committee show that instruction through correspondence is rapidly increasing in our leading colleges and universities, and that no subject can be taught by this method more successfully than mathematics. Some of the State universities are using this method of extending work to reach a large number of people within their respective States who desire mathematical training to assist them in their various vocations. In neither the universities and colleges nor the independent correspondence schools do we find uniformity in courses of study of requirements for admission to the courses or standards for securing evidence that courses have been completed. The courses in methods of the universities and colleges, with the exception named, follow more closely similar courses for residence students than do those of the independent schools, and reports from all institutions complying with the committee's request show that the work accomplished is of such nature as to be of permanent benefit to the student.

C. THE TEACHING OF MATHEMATICS IN SCHOOLS AND COLLEGES FOR NEGROES.

Source of data.—The following report is based largely upon material furnished by the following well-known schools and colleges for negroes:

Atlanta Baptist College, Atlanta, Ga.
Atlanta University, Atlanta, Ga.
Fisk University, Nashville, Tenn.
Florida Agricultural and Mechanical College, Tallahassee, Fla.
Shaw University, Raleigh, N. C.
Talladega College, Talladega, Ala.
Virginia Union University, Richmond, Va.
Hampton Institute, Hampton, Va.
Armstrong Manual Training School, Washington, D. C.
Colored High School, Baltimore, Md.
St. Augustine's School, Raleigh, N. C.

Character of the work.—Owing to the poor instruction in the elementary public schools generally, the schools of secondary and higher grades for negroes are compelled in most cases to offer elementary courses also. And even when the teaching happens to be good in the elementary school the chances are that the upper grammar grades are not given. In order, then, to connect their courses with the ordinary elementary-school courses it is necessary for the private schools to do some elementary work. The bulk of the work of high-school and college grade among colored people is done in these private schools. There are comparatively few high schools and colleges for colored youth maintained at public expense. Most of the private schools and colleges carry all of the elementary grades from the first through the eighth, and all of them have at least the last one or two grammar grades. In the best of these schools the work of a given grade is about the same as that in the corresponding grade in northern public schools, but the pupils are often considerably older. In the public schools the grades fall somewhat behind similar ones in the North.

The curriculum.—In the schools for negroes, then, the work in mathematics is about as follows:

Elementary school. Years I–VIII, inclusive. (Often only I–VII years are given in the public schools.) Five recitations per week, except in certain industrial schools like Tuskegee and Hampton with only three and four recitations per week. In the lower grades the periods are usually 30 minutes and in the upper at least 45 minutes.

Arithmetic usually is the only subject offered.

High (secondary) school.

IX. Algebra, 5 recitations per week.
X. Algebra, half year, 5 recitations per week.
Plane geometry, half year, 5 recitations per week.
XI. Plane geometry, half year, 5 recitations per week.
Solid geometry, half year, 5 recitations per week.
Advanced algebra, half year, 5 recitations per week.
XII. Mathematics rarely required.

The following are offered in a few cases:

Advanced algebra, half year.
Plane trigonometry, surveying, half year.
Analytical geometry, half year.
Mechanics, half year.
Mathematics of chemistry, half year.
Shop problems, half year.

Few of the schools give the work in mathematics in exactly the order given above. But nearly all of them devote about three years to the algebra and plane geometry ordinarily taught in a high-school course. Most of them require a year and a half for the elementary

algebra. For the plane geometry a year is usually given, but the schools vary all the way from a half year to a year and a half for this subject. A half year is the rather uniform time given to solid geometry. The latter subject and advanced algebra, however, are offered by but few institutions in their high-school departments.

College. The required work in mathematics usually covers only the first two years of the college course. Some choice of subject is often allowed in this period. But few schools offer other electives in subsequent years.

The following courses are commonly given:

College algebra, half year, 4 or 5 periods per week.
Solid geometry, half year, 4 or 5 periods per week.
Plane trigonometry, half year, 4 or 5 periods per week.
Spherical trigonometry (in a few cases).
Analytic geometry, whole year in a number of cases.
Surveying, half year.
Astronomy, half year.

Most of the schools do not offer analytic geometry or spherical trigonometry. But several of them not only give these subjects, but differential and integral calculus also as electives. Such schools are Howard University, Fisk University, Virginia Union University, Atlanta University, and possibly some others.

The work in arithmetic in the grades.—The work in arithmetic is usually routine and conventional, following the textbook, with little regard for the choice of subject matter. Some of the better schools are beginning to eliminate obsolete material and to leave such subjects as square and cube root for work in algebra. Too often by far the teaching is abstract and usually without practical applications. The most notable exceptions to these conditions are to be found in the model schools and teacher-training departments of the better schools, such as Spelman Seminary, Atlanta University, and the Whittier School, at Hampton Institute. With the work of these primary schools should be mentioned also that of industrial schools like the Calhoun Colored School, the Penn School, Tuskegee Institute, and Hampton Institute. In such institutions the arithmetic work is very concrete and the method of teaching mainly objective. Lines, solids, paper cutting, measures and weights are used in the teaching, and outdoor measurements are made by the students. Material for classroom exercises is taken from the fields, shops, offices, and industrial activities of the schools and their neighborhoods. And classes are taken into the shops and offices to see practical illustrations of the problems studied.

Secondary work.—The work in mathematics in the secondary or high schools for negroes corresponds favorably in subject matter with that of the ordinary northern high school. White teachers from the

North or colored teachers educated in northern schools have charge of most of this work.

The algebra covers at least elementary work, including ratio and proportion, radicals, radical equations, theory of exponents, and the binomial theorem, and in the stronger schools special drill in quadratics; complex numbers, logarithms, and the use of the graph.

The plane geometry is usually taken complete with original demonstrations, but in many schools a number of the original exercises are omitted.

Solid geometry is taken complete.

The public high schools of Washington, D. C., offer perhaps a fuller course than any others of the secondary schools for negroes. In addition to the algebra and plane geometry usually given they have advanced algebra, plane trigonometry and surveying, solid geometry, plane analytical geometry, mechanics, mathematics of chemistry, and shop problems.

In these schools the teachers of algebra frequently visit the grades where arithmetic and elementary algebra are taught, so as to keep their work in close touch with that of the grades. Pupils coming from the grades find the transition less difficult for this reason.

In geometry pupils are required to make accurate geometrical constructions of figures embodying the conditions of the hypothesis for each theorem. Simple original exercises are begun early in the course and continued throughout. Particular attention is given to the development of general principles for attacking original problems. And the lesson assignments always include preparation for advance work.

The work in solid geometry begins with preliminary instruction in the best conventional ways of representing in drawings intersecting and parallel planes, and with a review of the idea of perspective in the representation of solid figures. An easy and rapid free-hand oblique projection is taught and insisted upon in the construction of all figures. Construction of cardboard models is encouraged at the beginning of the course.

In trigonometry the aim is to complete the simplest and most concrete part of the subject before taking up the more general and abstract parts. In the latter part of the course surveying is stressed, and the derivation of many of the formulas is reserved till some problem in surveying shows an actual need of them.

This work is fairly typical of what is done in the more advanced high schools and colleges with secondary grades. In the weaker institutions and in the industrial schools, like Tuskegee and Hampton, no courses above plane geometry are offered. In the latter schools particularly, the work in geometry is largely inventional. Many of the most important theorems are demonstrated and a good deal of

original work is done. This study supplements the courses in mechanical drawing. Its relations to the various trades taught are constantly shown.

Practical applications.—In arithmetic, problems of a practical character are used by the better institutions, as already mentioned. Geometry is used in mechanical drawing and in the work of the various trades. Mathematics is also applied in the work in chemistry, physics, and in the electrical engineering laboratories of some of the colleges, in such high schools as those in Washington and possibly in the new and finely equipped Sumner High School in St. Louis, the most complete public high school for colored youth in America, costing, with equipment, $500,000. The schools for colored students are, however, not strong in applied mathematics. As President John Hope, of the Atlanta Baptist College, well says: "Openings in jobs that call for higher mathematical development have been almost absolutely closed to young negroes, so they have not had the incentive that white students have had along mathematical lines."

College Work.

Character of the work.—Apart from their weakness in applied mathematics, the colleges for colored youth do about the same kind of work as the northern colleges. They naturally do not offer so many courses. But they very uniformly give those indicated above. And they do their work with considerable thoroughness. The better students from the best of the schools are able, upon completing their courses to enter the junior classes at Harvard and Yale, and a few have even been able to enter the senior classes. Many of the northern universities give them full college credit for their work in mathematics. In fact these students are more likely to compare favorably with northern white students in mathematics than in the natural and social sciences. For the latter the colored schools are not well equipped. Generally, as Shaw University reports, the graduate of the preparatory department of these schools is about a year below the standard of the typical high school, and consequently the college classes are below the standard to the same extent. There are, of course, exceptions and especially in the case of a few high schools in the border States.

Entrance requirements.—Since practically all these schools carry the elementary grades to some extent, there is no hard and fast requirement for admission to the institution. But the work of each department must be completed before a student is admitted to the next higher department. None of these institutions is more exacting in such matters than Atlanta University, which writes:

For entrance to the high school we require an examination in the arithmetic of the eighth-grade grammar school. The inability of many negroes, especially in the

country, to get good preparation forces us to be very lenient in these examinations; and a large percentage of our pupils have to spend two years in the completion of the work required of the first year.

For college entrance we require an examination in the mathematics of our high-school course or its equivalent.

This, for Atlanta University, means algebra, including quadratics, logarithms, and the use of the graph. Most colleges are not quite so rigid in their requirements.

For entrance to Hampton Institute proper, work that would prepare for a typical fifth grade is required. Tuskegee's requirements are no higher.

For the well-ordered city high schools the usual work of seventh or eighth grade is required for admission.

Requirements for graduation.—Very few of the schools offer more than one or two electives in mathematics. So that with the exception of spherical trigonometry and analytic geometry, and in a few cases surveying and astronomy, all the work set down above is required in the several departments as a rule.

The mathematical ability of the colored race as compared with the white race.—The general feeling in the institutions covered by this report is that the differences between the races in the matter of mathematics, in so far as any differences exist, are due to conditions rather than to race characteristics. In addition to this, it is interesting to notice the opinions given directly by the 11 schools contributing to this study. Five of them feel that there is no difference due to race. Two are of the opinion that colored students generally are not the equals of white students. One school was not able to make any comparison. Three did not reply to the question regarding a comparison of the races. Replies were made by both white and colored teachers who have had years of experience in colored schools and in some cases in white schools.